



International School
on
Aperiodic Crystals

**Examples of incommensurately
modulated structures and
composites studied using
transmission electron
microscopy**

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Purpose of this lecture

At the end of this lecture you should be able to:

- Understand the TEM paragraph in papers about IMS and CS
- Be able to make solid comments about conclusions claimed from TEM by different sources (collaborators, papers,...) by knowing some possible pitfalls
- Decide whether it would be useful to do TEM on your own IMS or CS
- Make basic interpretations of TEM data on your own materials by yourself

Outline of the lecture

The three most frequently used techniques in case of IMS/CS:

- Electron diffraction
- HAADF-STEM
- HRTEM

ED: the main differences with XRD



- Shorter wavelength, almost flat sections through reciprocal space
- A nanometer size particle is enough
- Multiple diffraction
- Can be combined with direct imaging and compositional analysis

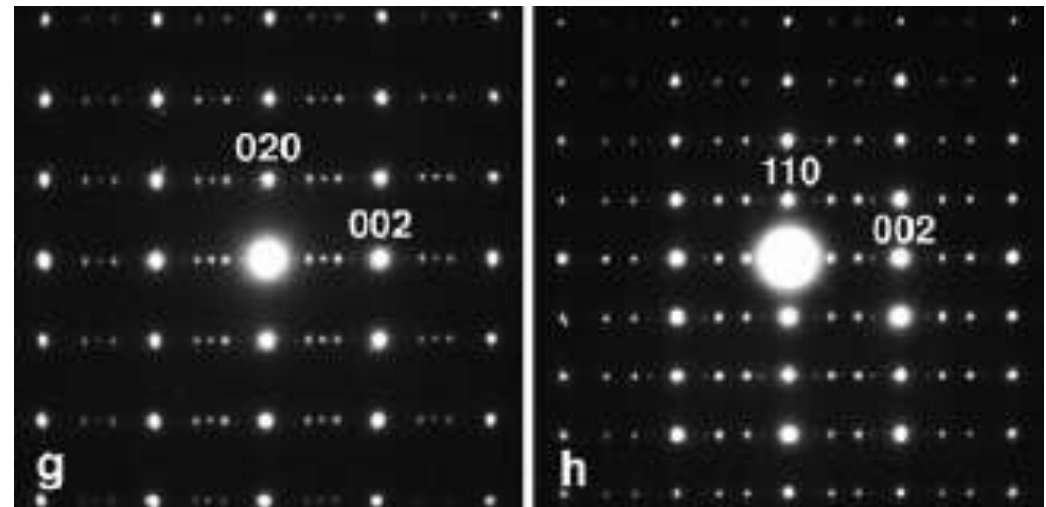
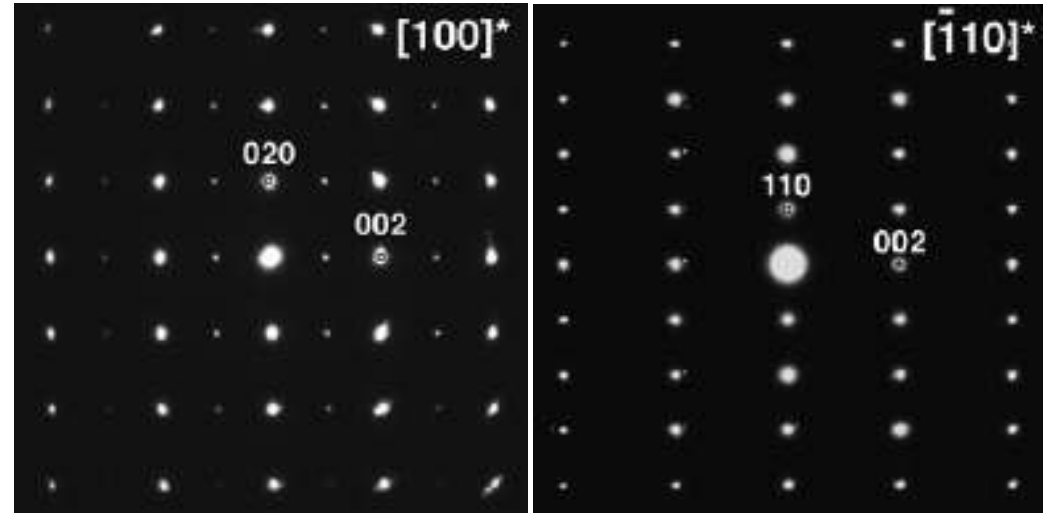
Appearance very much like single crystal XR patterns



Incommensurately modulated structure example



Related commensurate structure

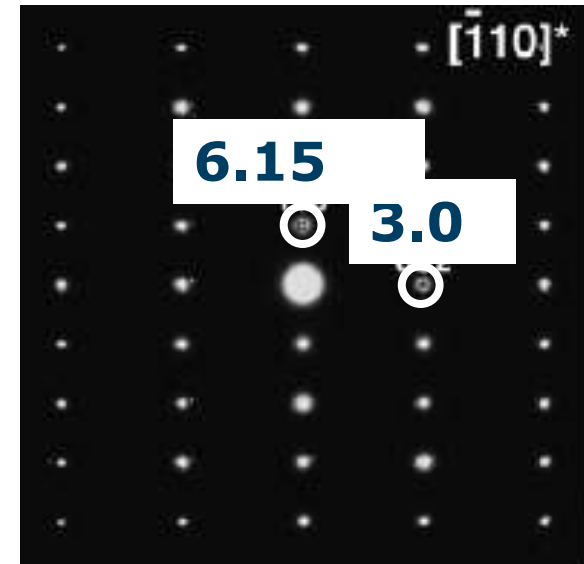
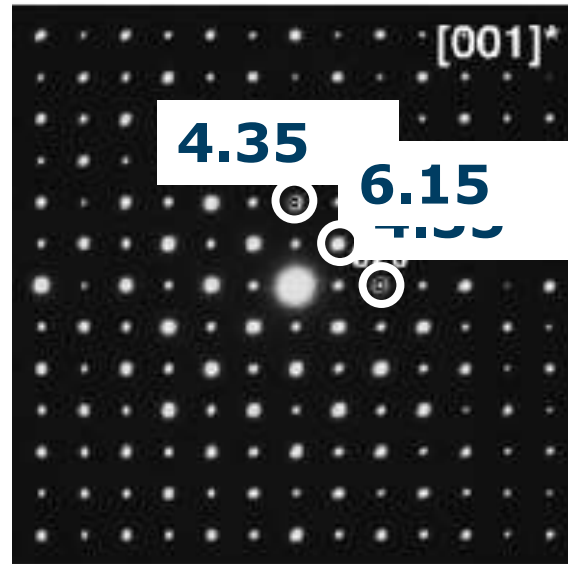
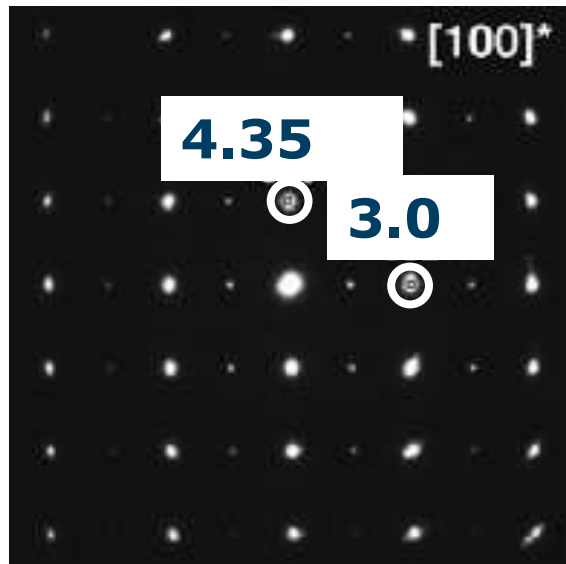


Abakumov et al., Chemistry of Materials, 17, 5, 2005, 1123-1134



IMS: indexing the subcell reflections

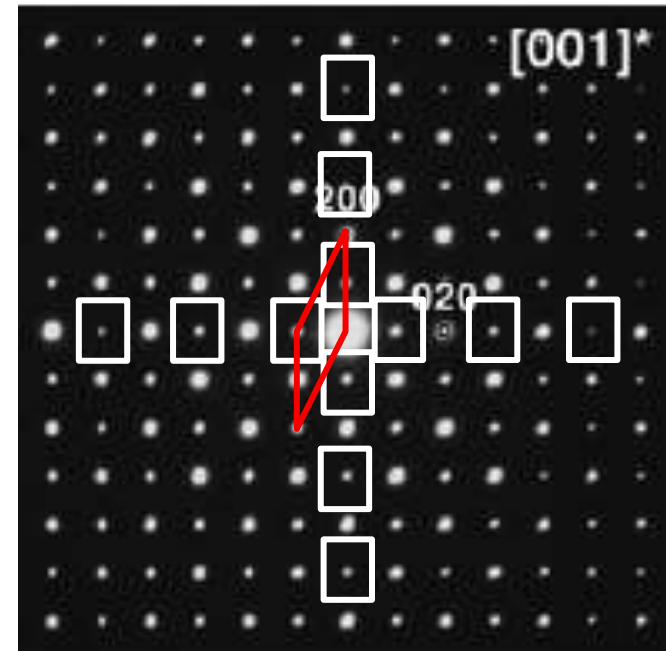
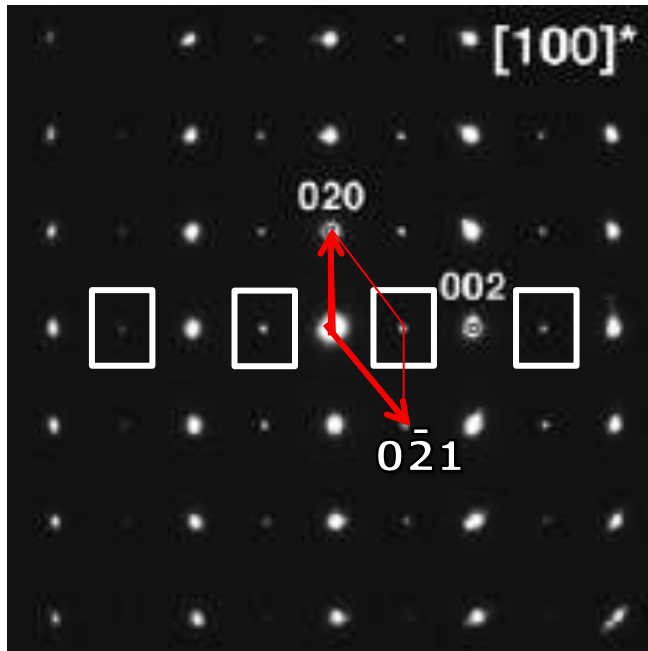
- Related commensurate structure Sb_2MnO_4
- $P4_2/mbc$; $a=b=8.7 \text{ \AA}$, $c=6.0 \text{ \AA}$
- Measure R , calculate d with $R \cdot d = C$ (know instrument constant)
- Index each reflection according to list of d -values for each reflection (cf. XRD)



All distances in \AA

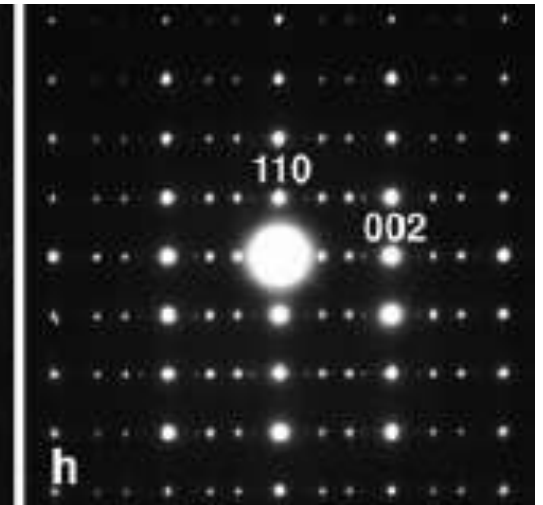
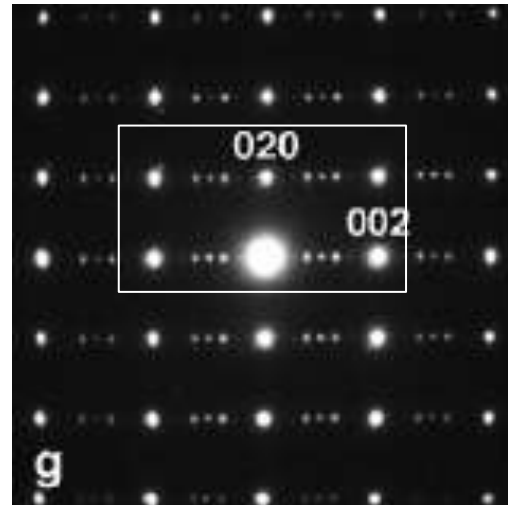
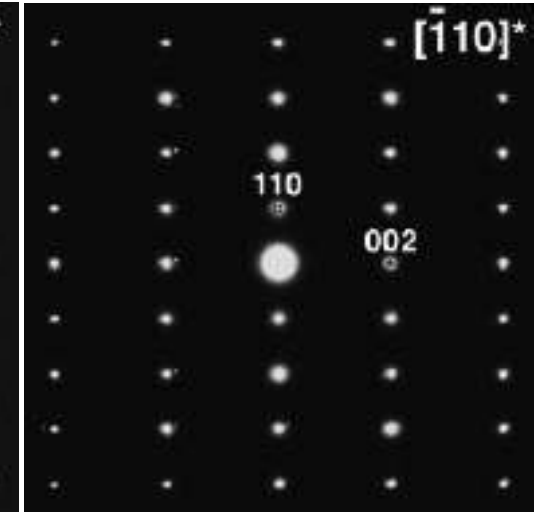
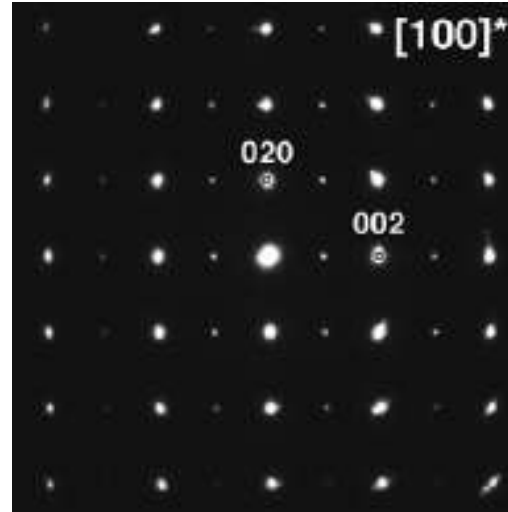
Dangers of double diffraction!

- $P4_2/mbc$
- Reflection conditions: $0kl: k=2n; hhl: l=2n; 00l: l=2n; h00: h=2n$
- In contradiction: $001, 003, \dots, 100, 300, \dots, 010, 030, \dots$ all caused by double diffraction



- Tilt around the row: forbidden reflections will disappear

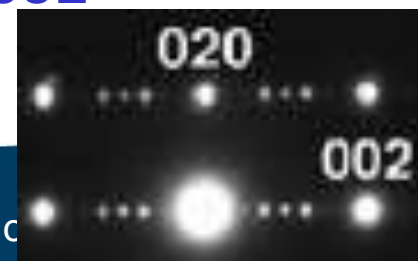
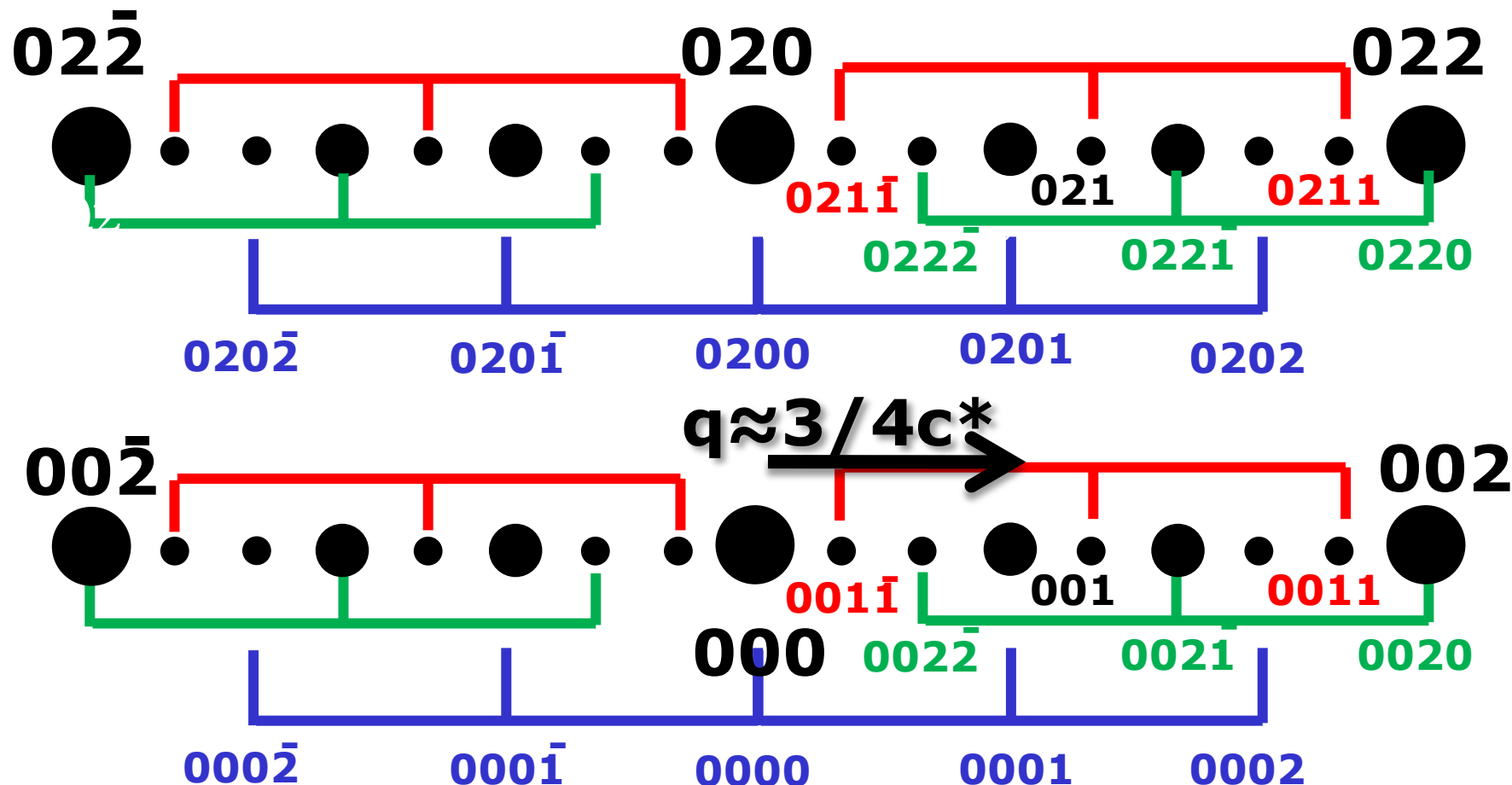
Subcell vs. IMS



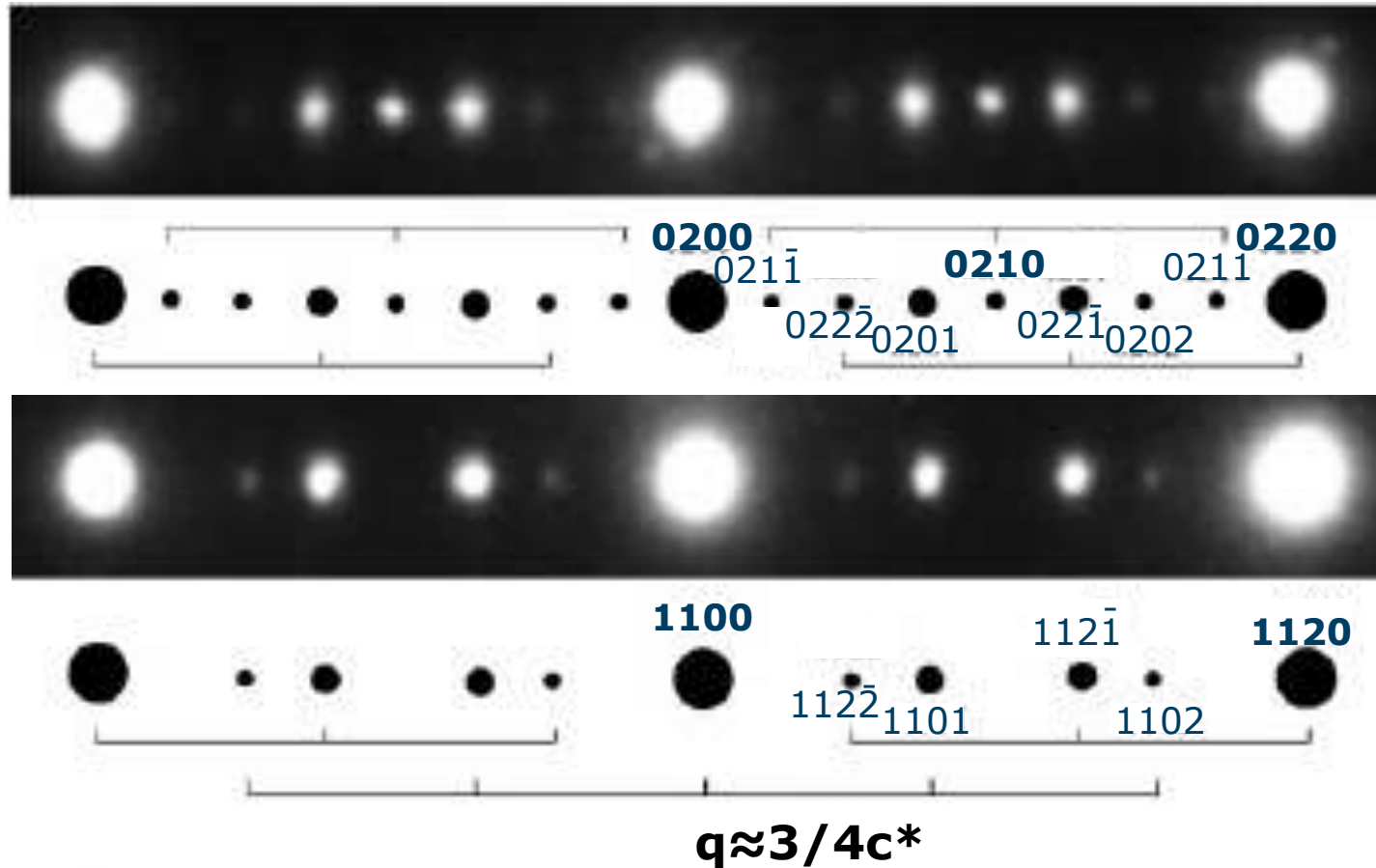
Abakumov et al., Chemistry of Materials, 17, 5, 2005, 1123-1134



Indexing the IMS



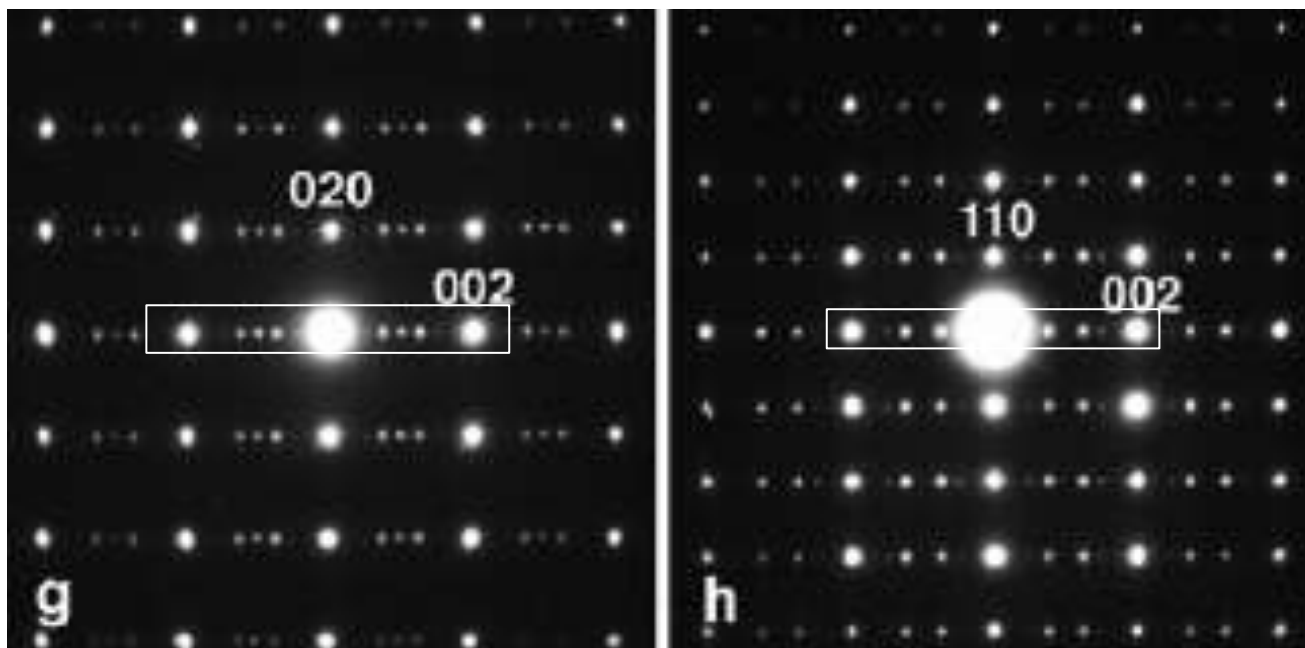
Reflection conditions for m



For this material: no special conditions for m: if subcell reflection is seen, also hklm with m=1 and m=2 is seen.

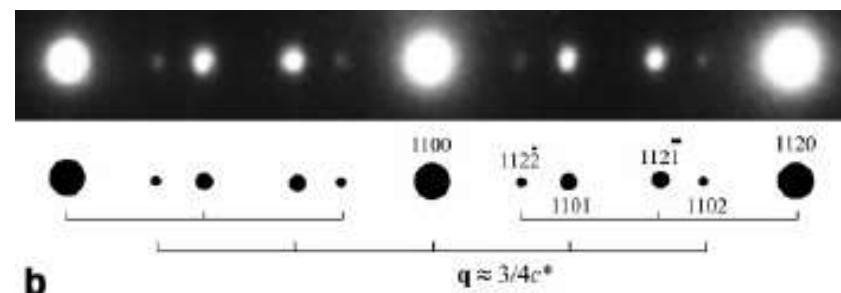
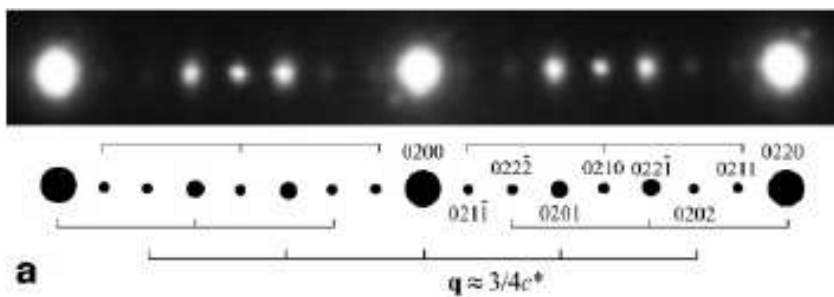


Reflection conditions



$$\left. \begin{array}{l} P4_2/mbc \\ 0kl: k=2n \\ hhl: l=2n \end{array} \right\}$$

$$\begin{array}{l} 0klm: k=2n \\ hhlm: l=2n \\ P4_2/mbc(00\gamma) \end{array}$$



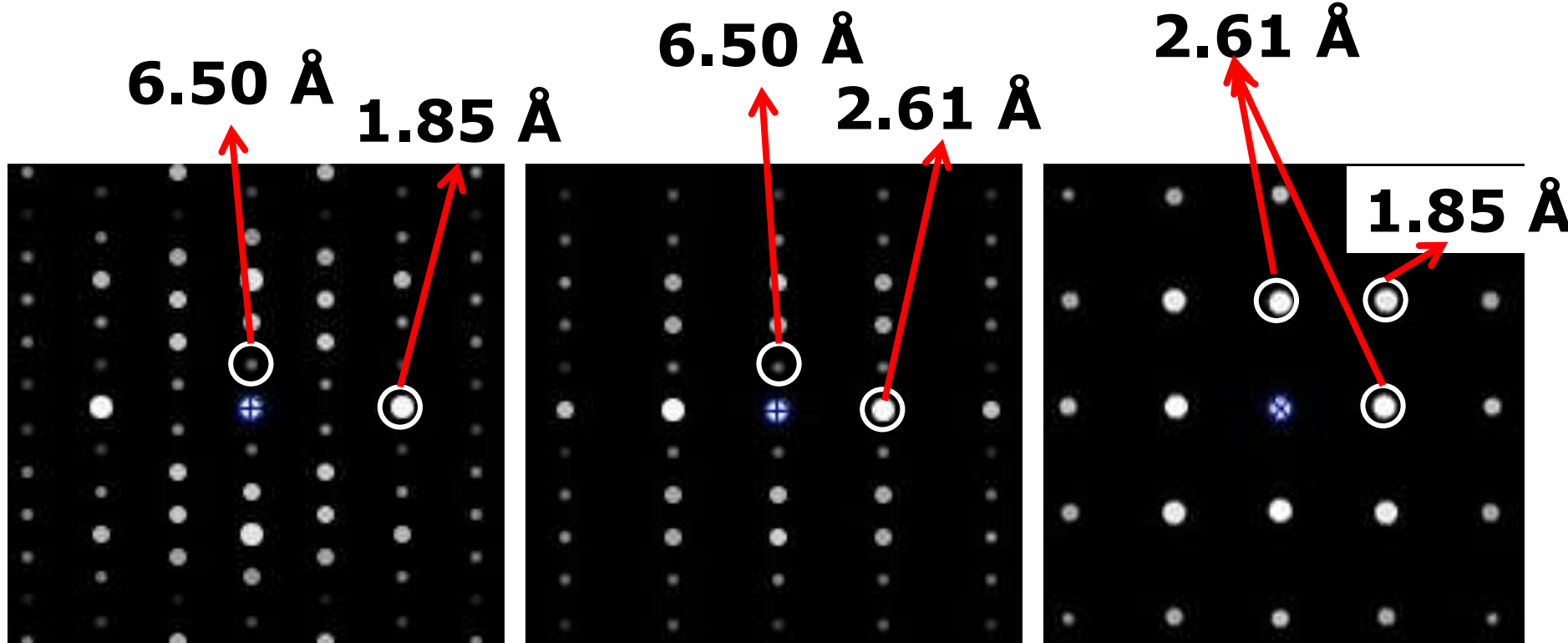
Exercise 1-1: IMS: index the non-modulated phase



- Index the commensurate structure LaSrCuO_4 , this will help you with indexing the next, incommensurate one.

Given data:

cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.



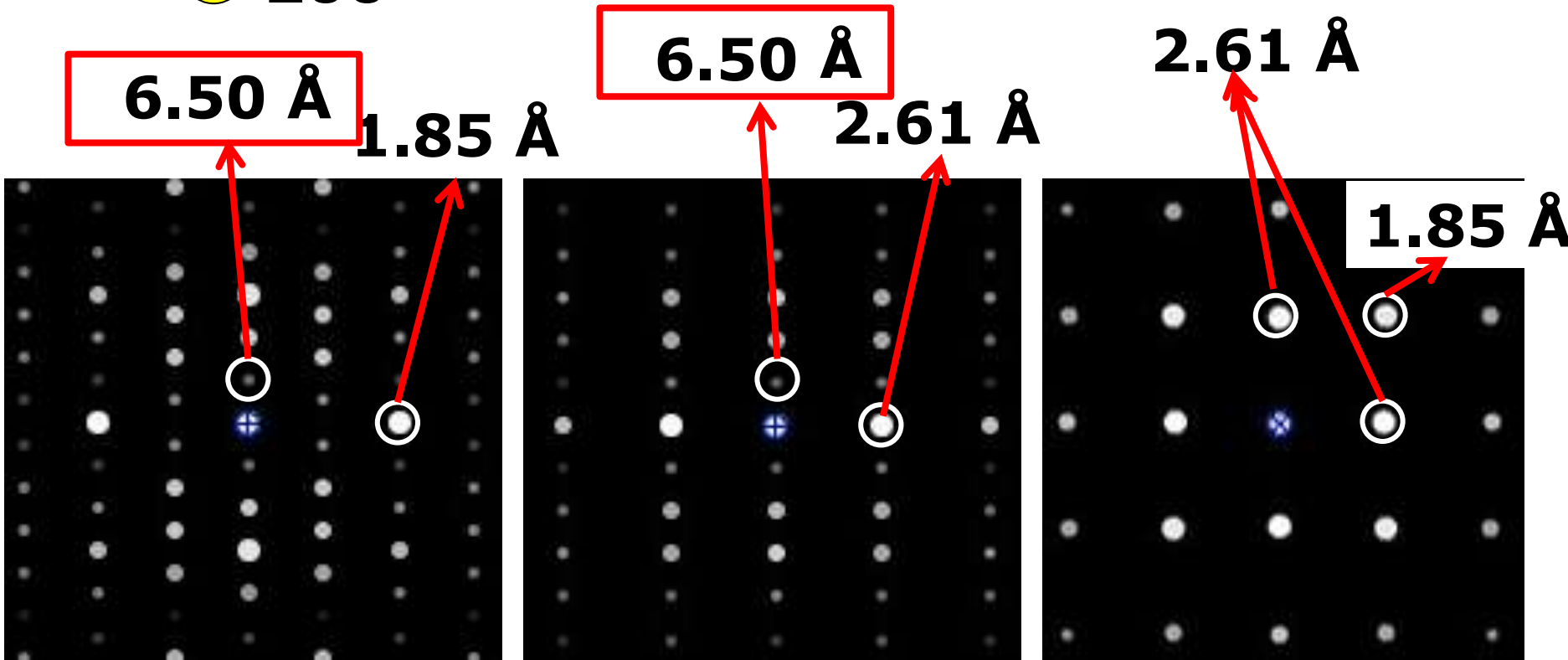
(Simulated ED patterns)



Given data:

cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.

- 001
- 002
- 200



(Simulated ED patterns)

Given data:
 cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.

 **001**

 **002**

 **200**

6.50 \AA

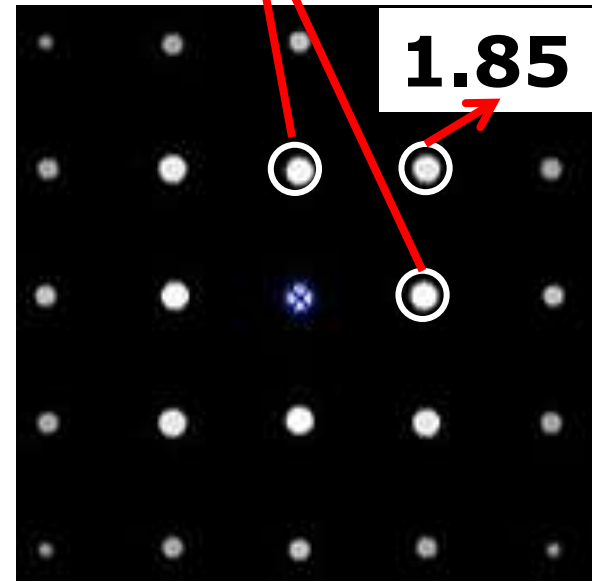
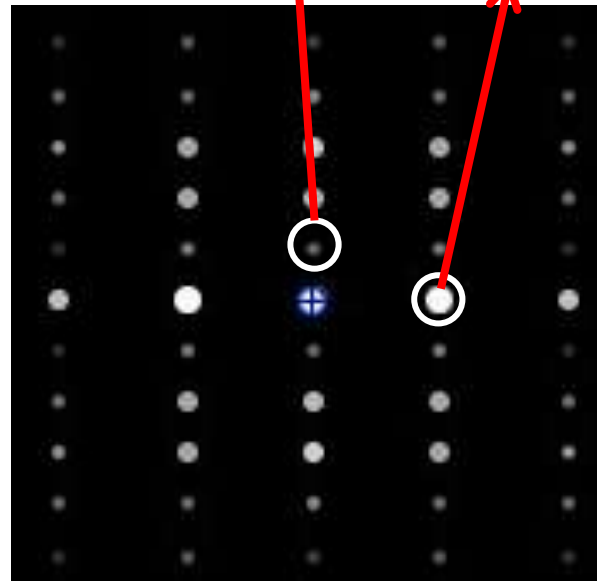
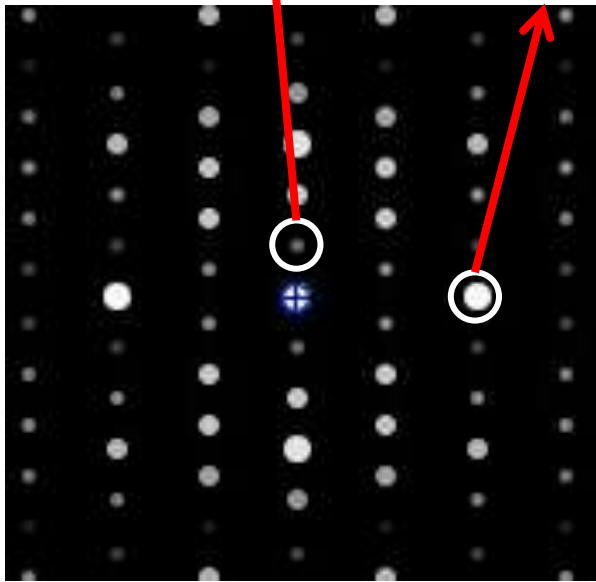
1.85 \AA

6.50 \AA

2.61 \AA

2.61 \AA


1.85 \AA



(Simulated ED patterns)

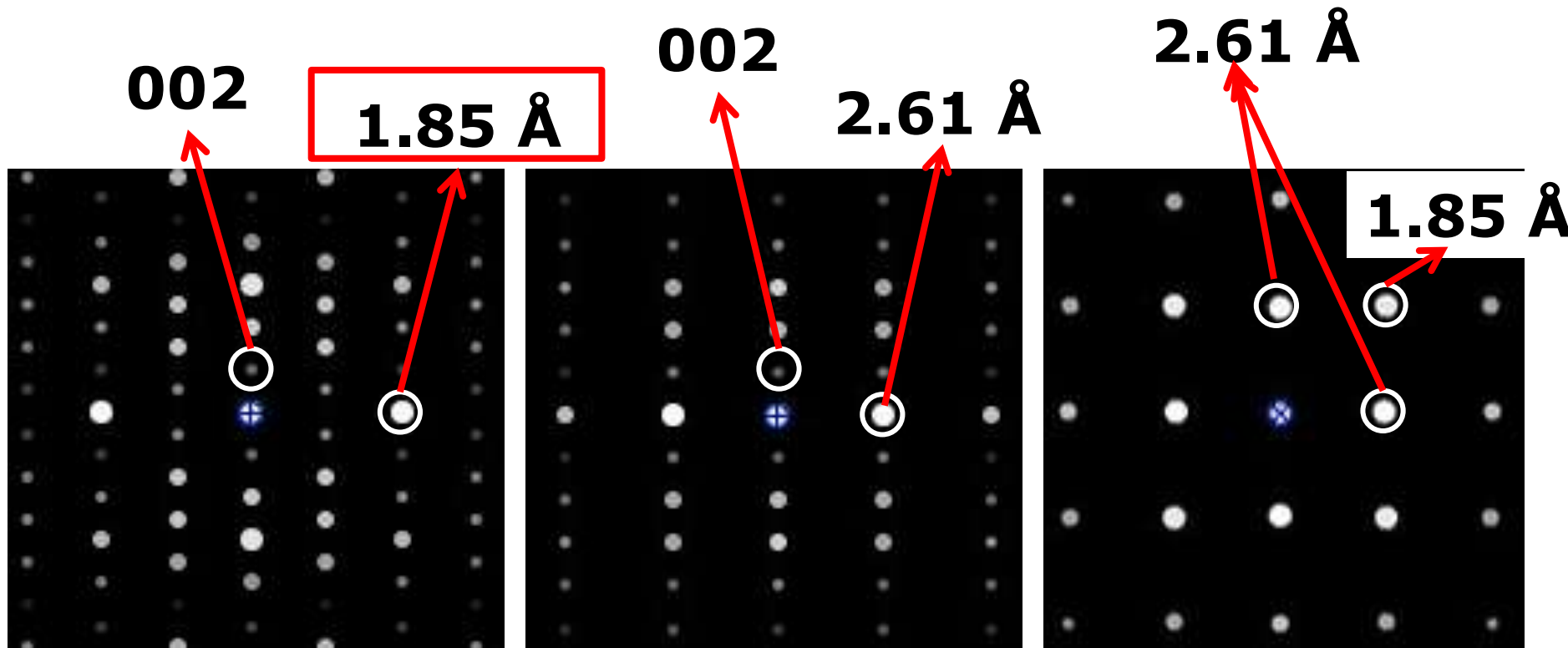
Given data:

cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.

 **010**

 **020**

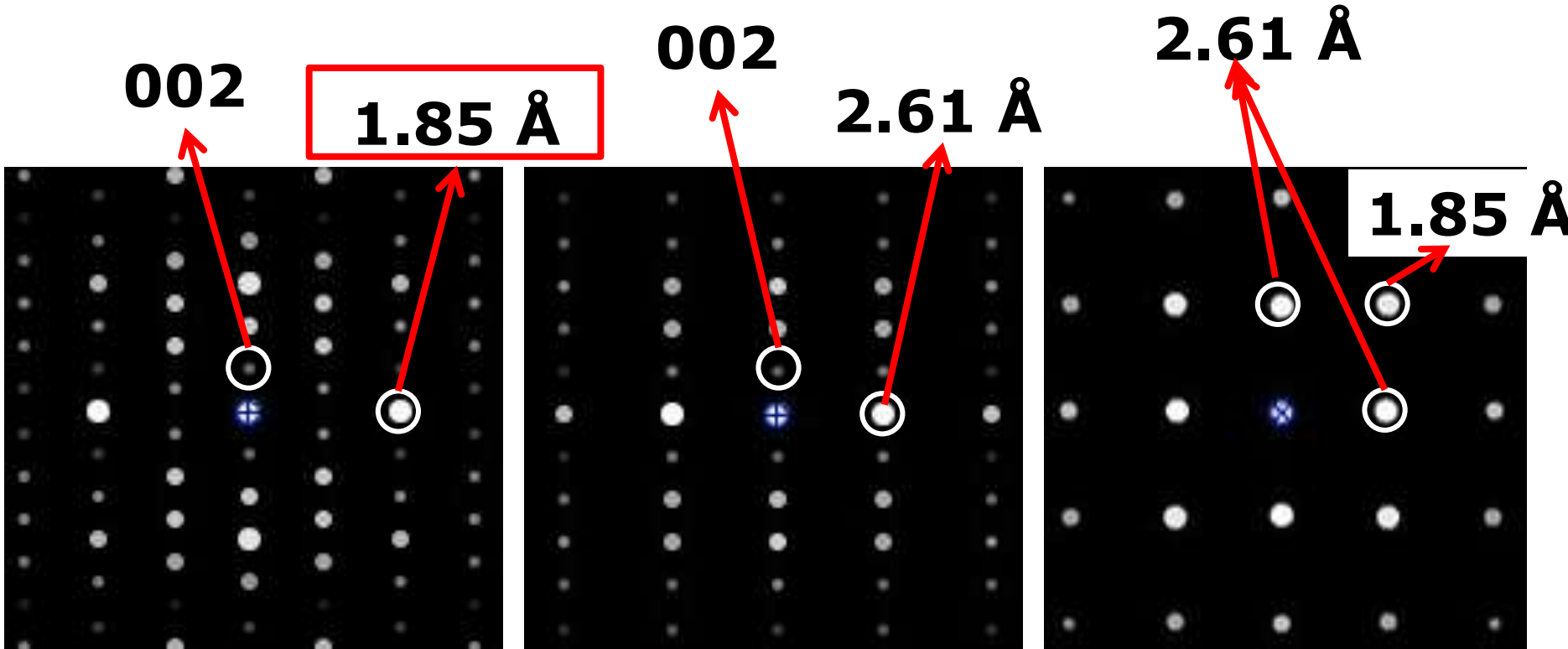
 **110**



(Simulated ED patterns)

Given data:
 cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.


- 010
- 020
- 110



(Simulated ED patterns)

Given data:

cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.

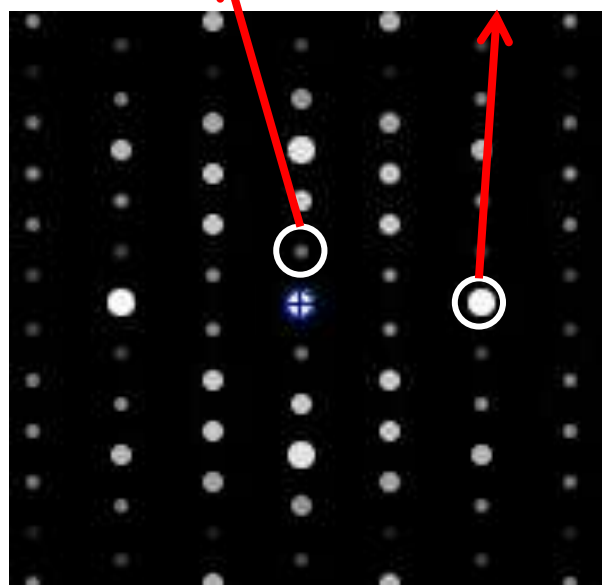
 **010**

 **020**

 **110**

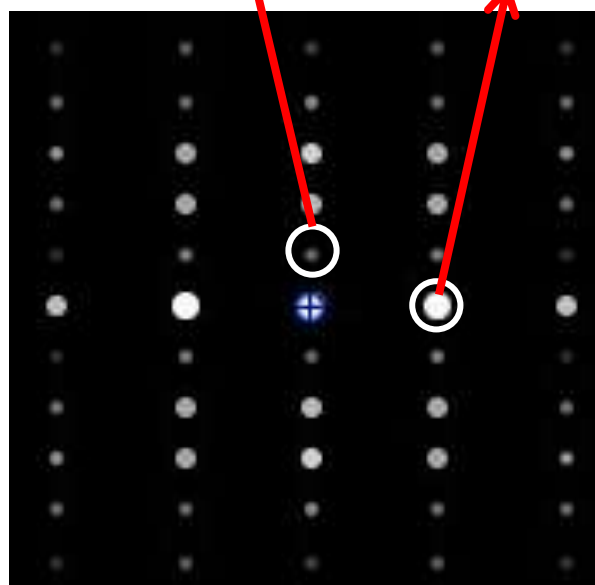
002

020



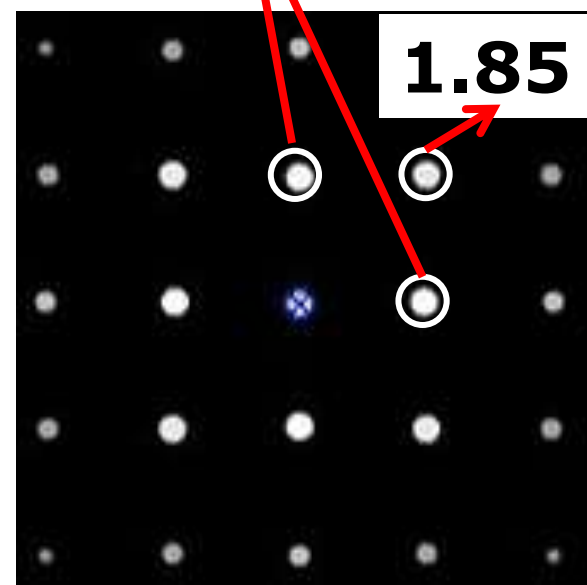
002

2.61 \AA



2.61 \AA

1.85 \AA



(Simulated ED patterns)

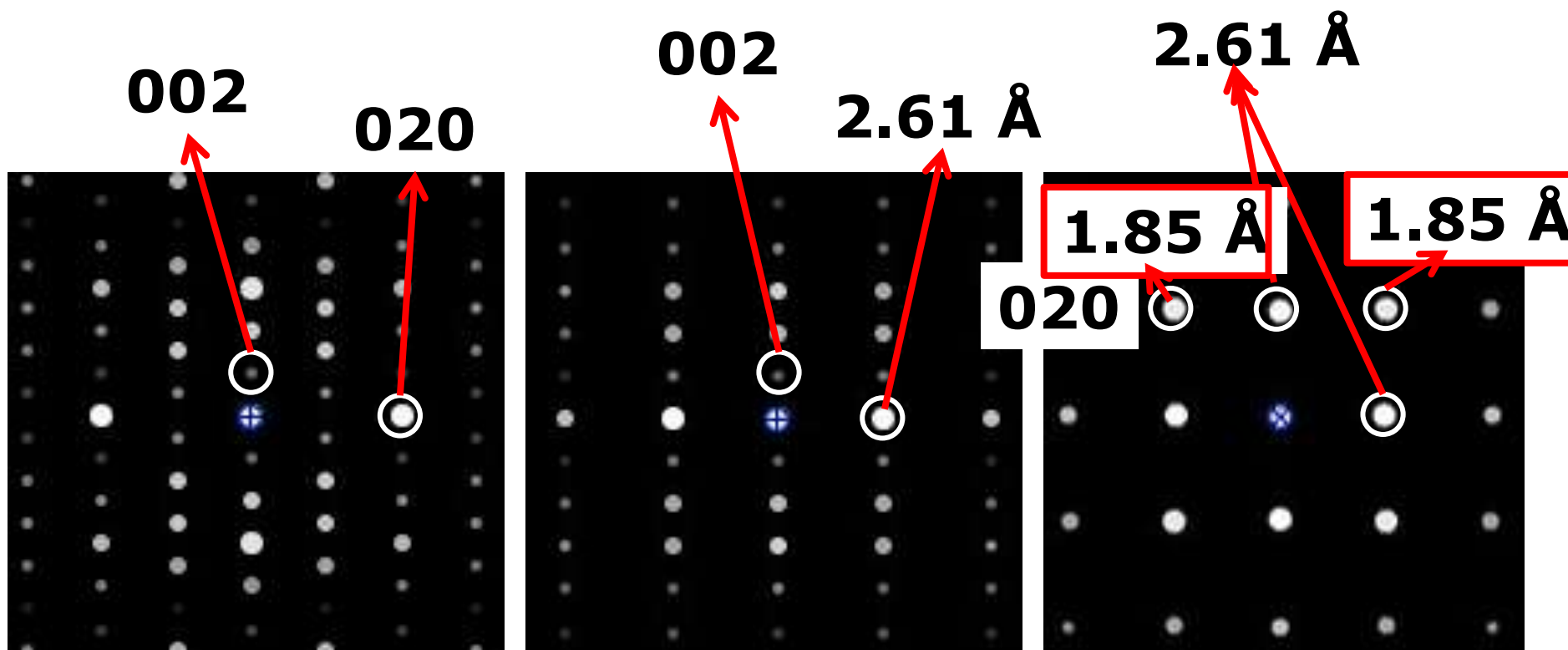
Given data:

cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.

 **100**

 **200**

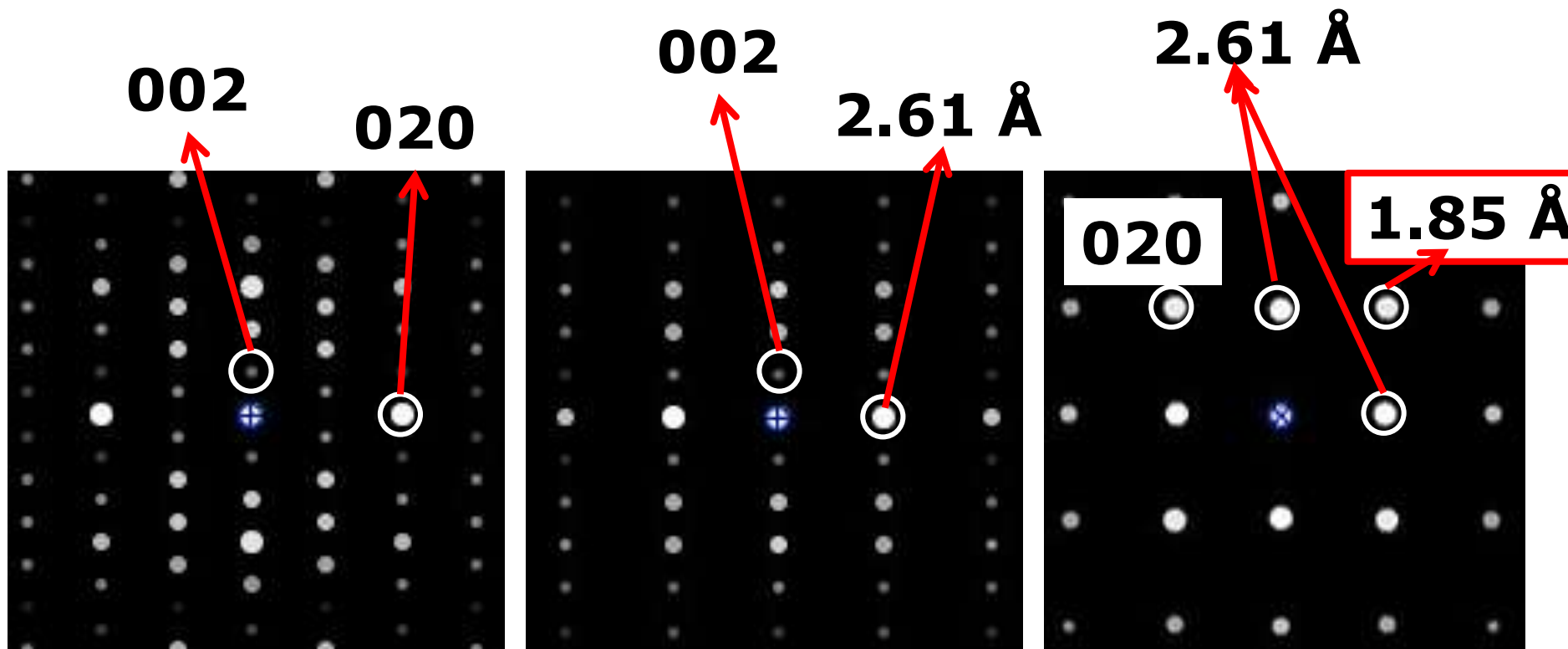
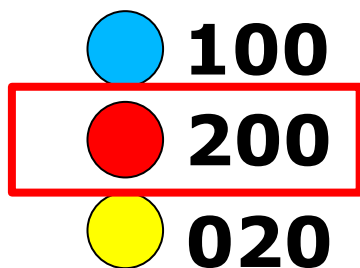
 **020**



(Simulated ED patterns)

Given data:

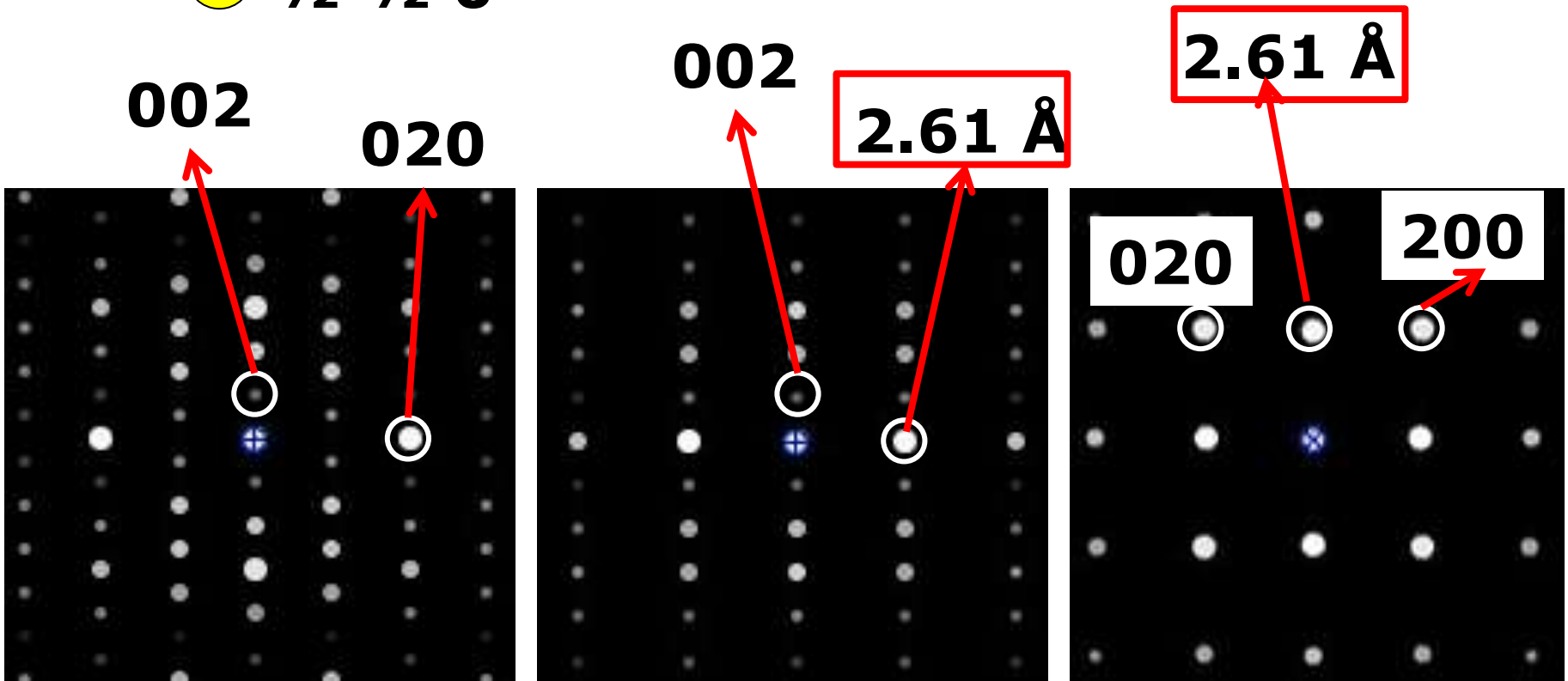
cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.



(Simulated ED patterns)


Given data:
cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.

- **110**
- **220**
- **$\frac{1}{2} \frac{1}{2} 0$**




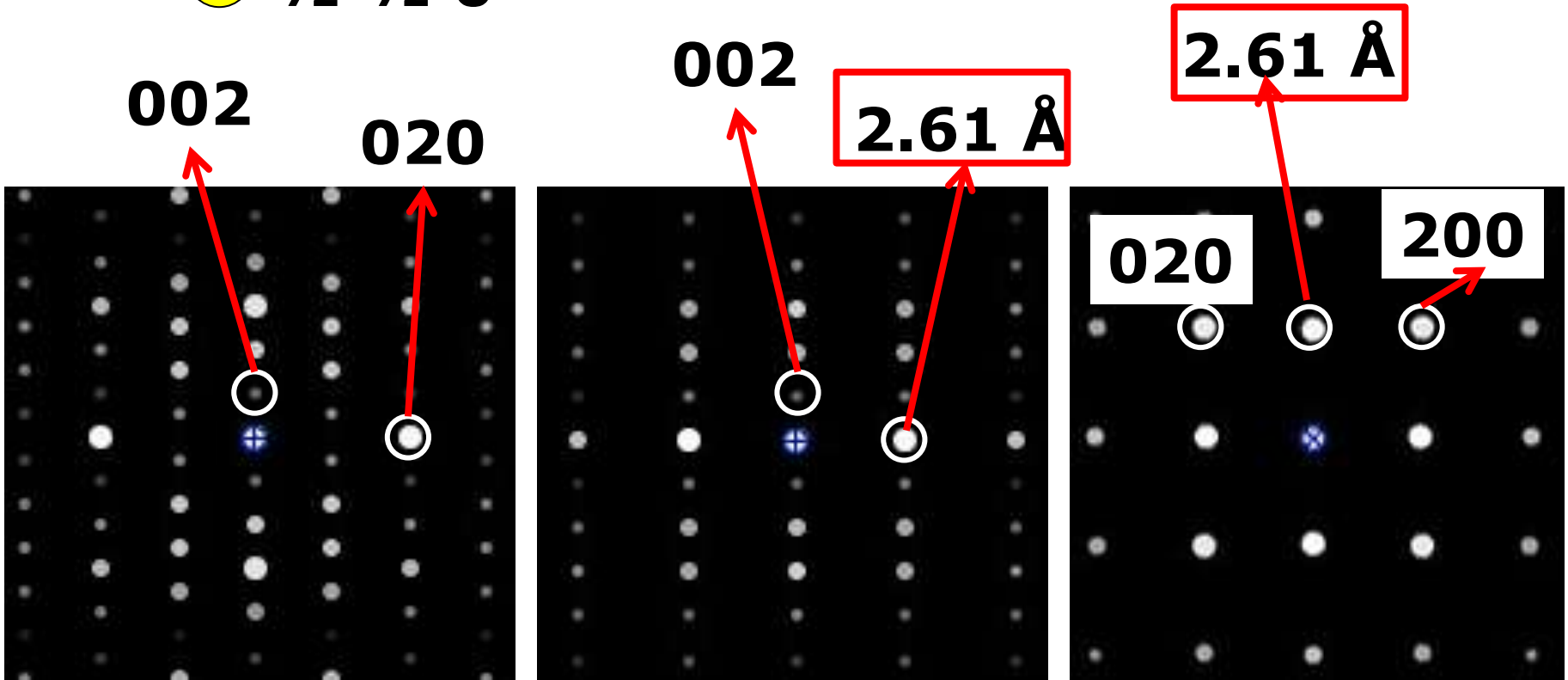
(Simulated ED patterns)

Given data:
 cell parameters of LaSrCuO_4 : $a=b=3.7 \text{ \AA}$, $c=13 \text{ \AA}$.

 **110**

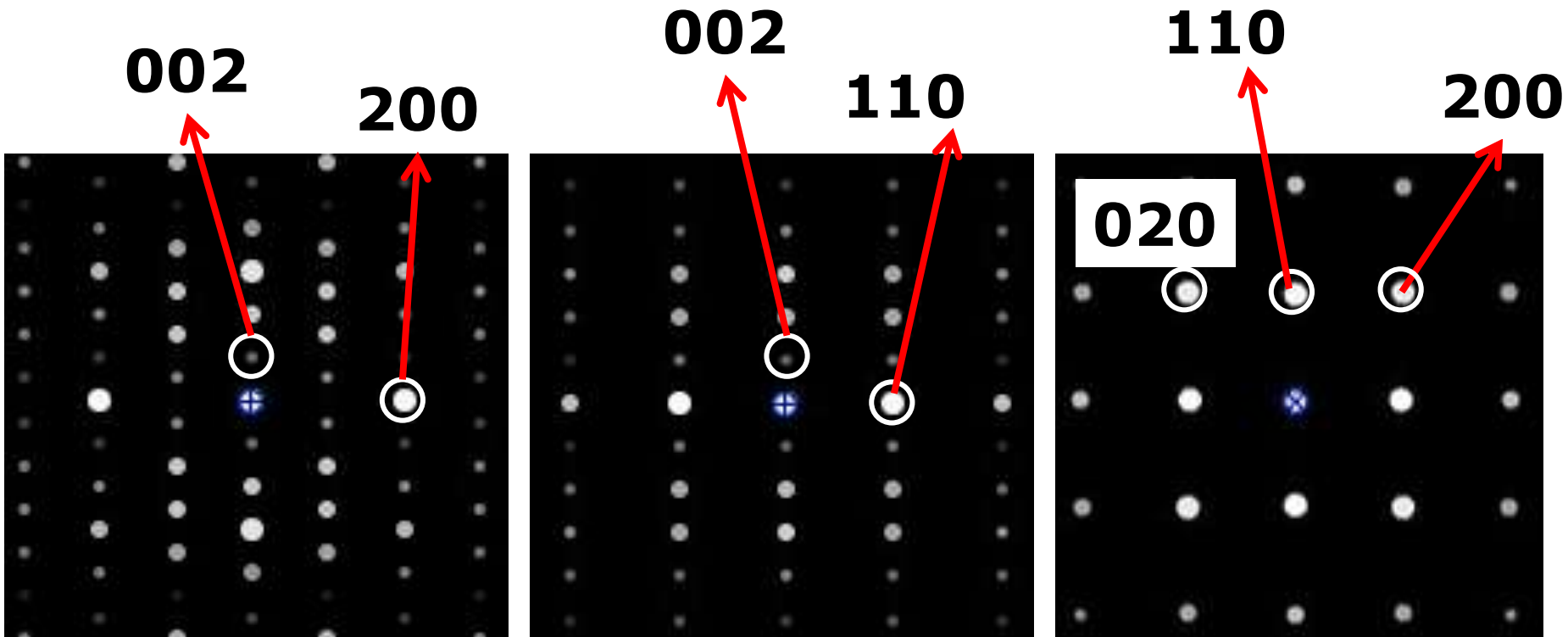
 **220**

 **$\frac{1}{2} \frac{1}{2} 0$**

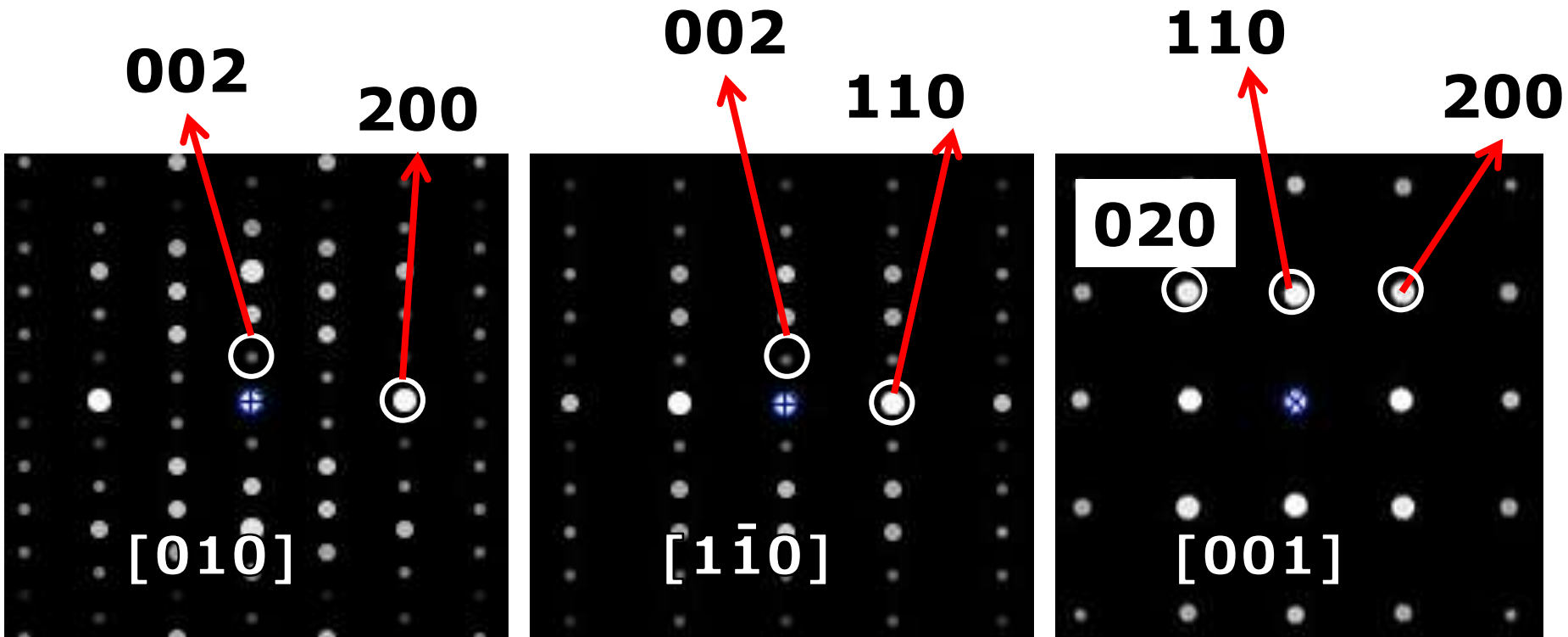


(Simulated ED patterns)

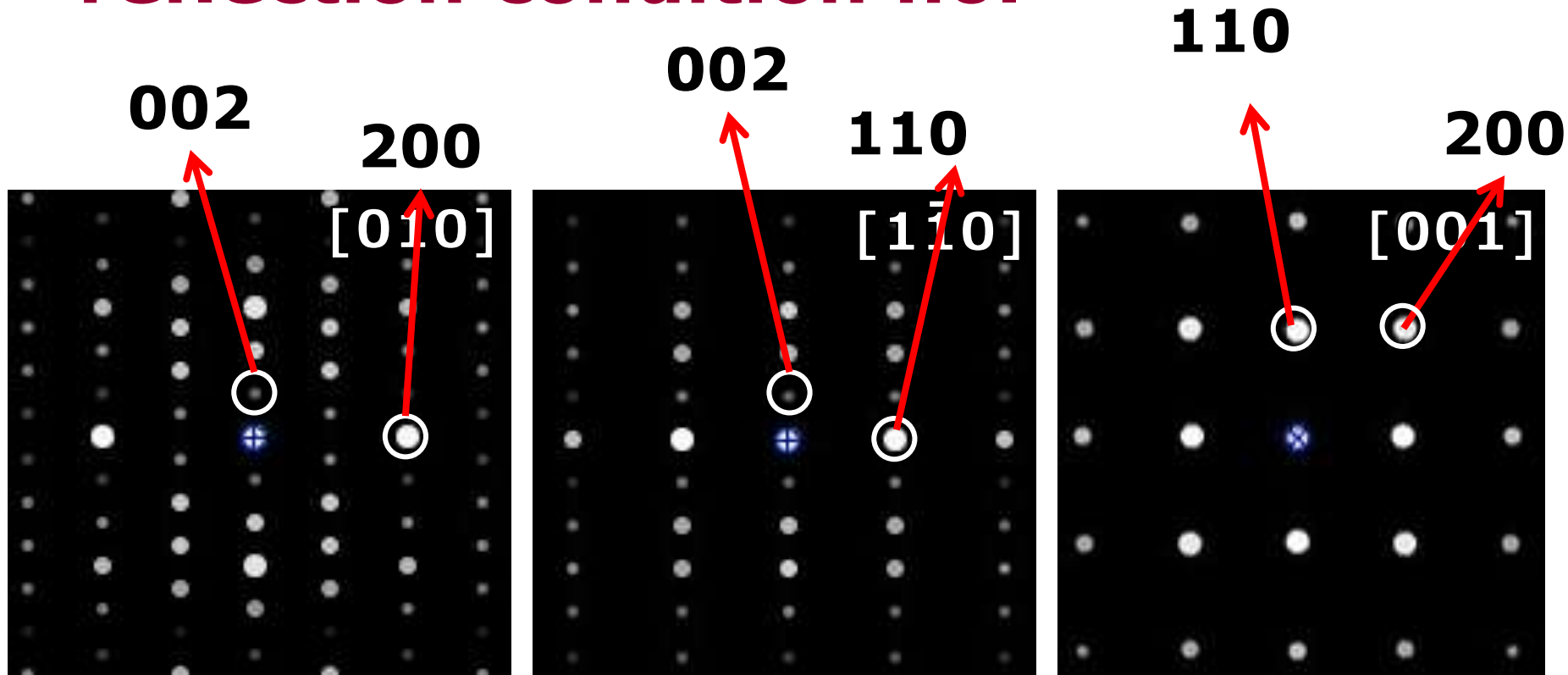
Indexed solution



Zone indices



Exercise 1-2: IMS: determine the reflection condition $h0l$

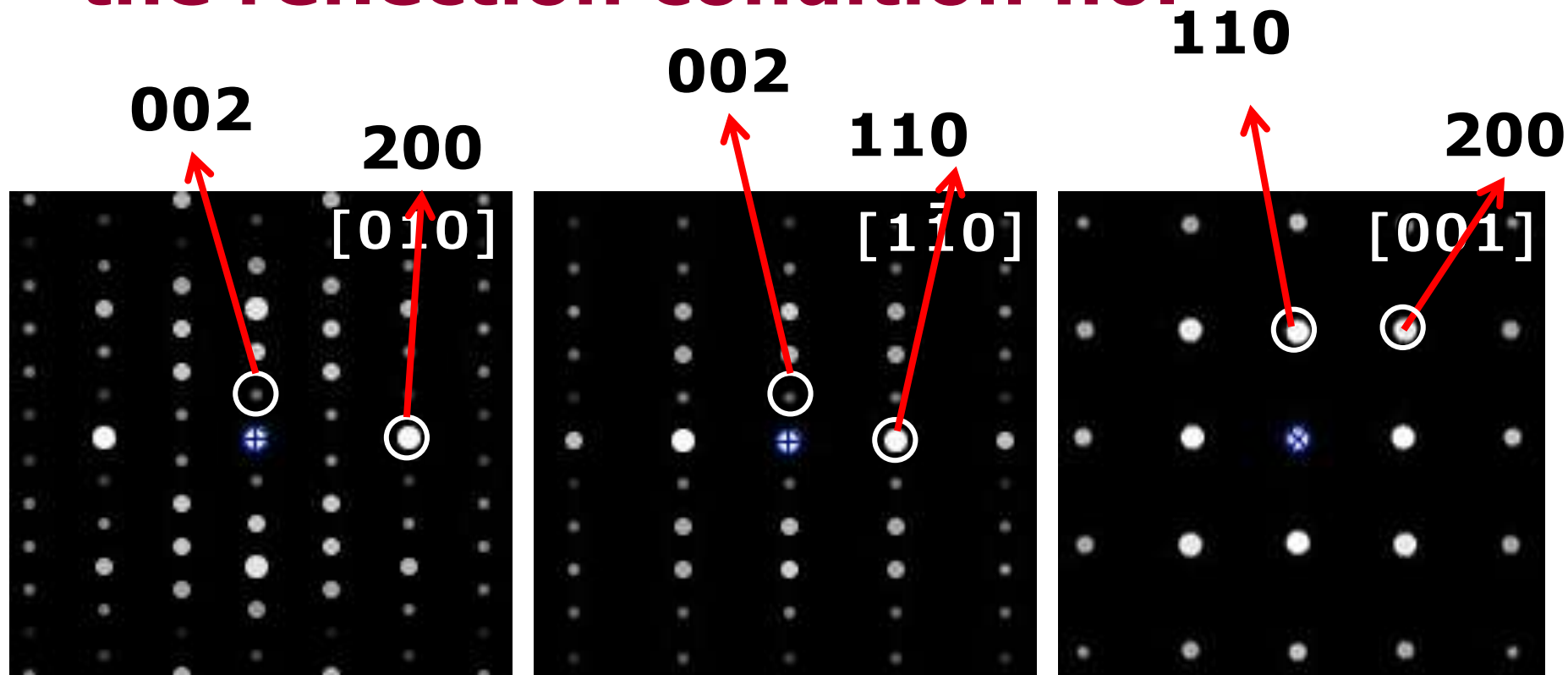


$h0l$: $h+l=2n$ ●

$h = 2n$ ●

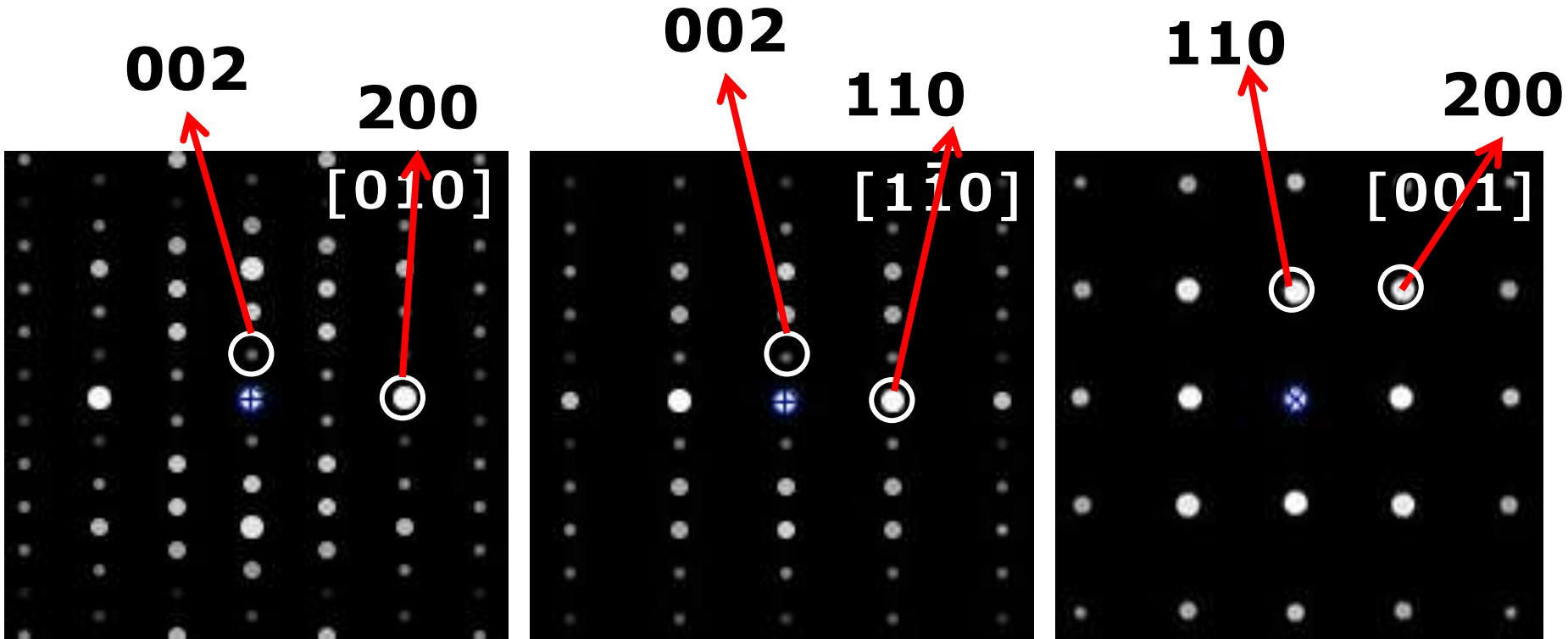
$l = 2n$ ●

Exercise 1-2: IMS: determine the reflection condition $h0l$



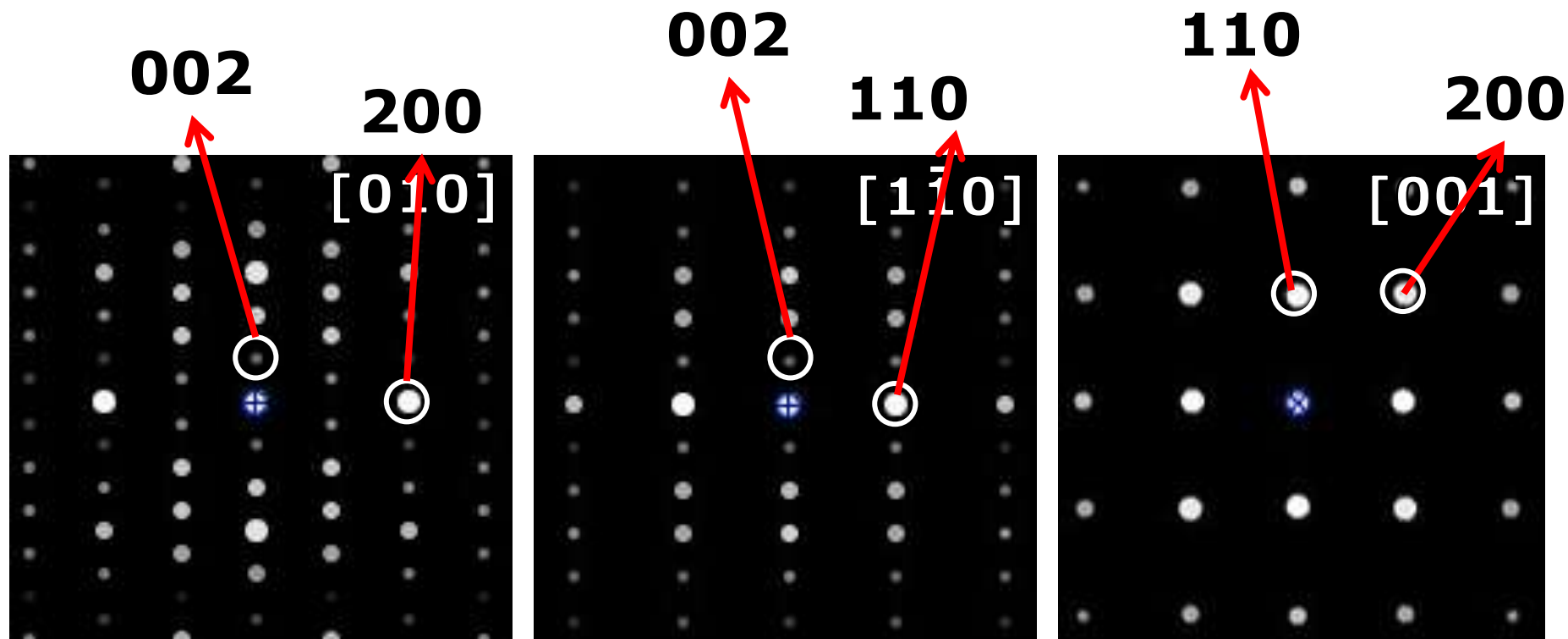
$h0l: h+l=2n$ ●

Exercise 1-2: IMS: determine the reflection condition hhl



$h0l: h+l=2n$ $hhl: h+l=2n$ ●
 $h = 2n$ ●
 $l = 2n$ ●

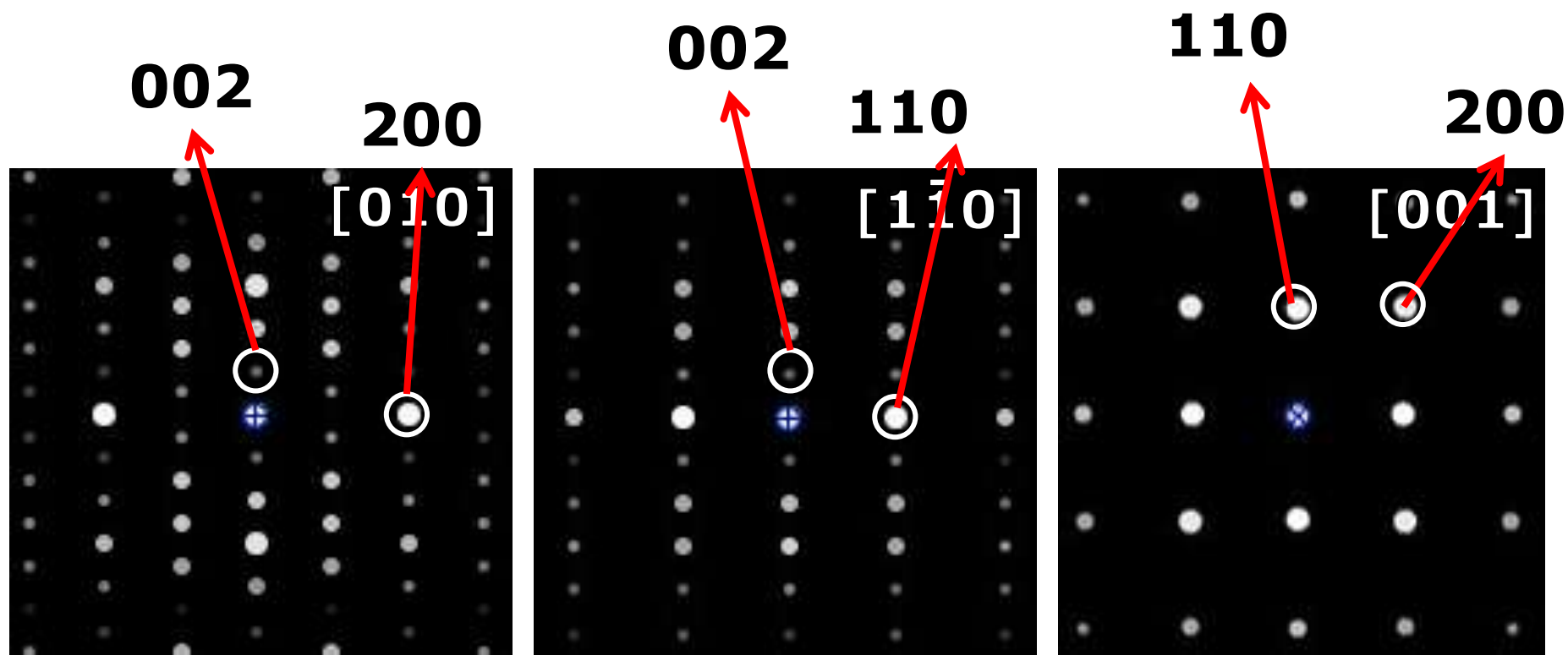
Exercise 1-2: IMS: determine the reflection condition hhl



h0l: $h+l=2n$ hhl:

$l = 2n$ 

Exercise 1-2: IMS: determine the reflection condition $hk0$



$h0l: h+l=2n$

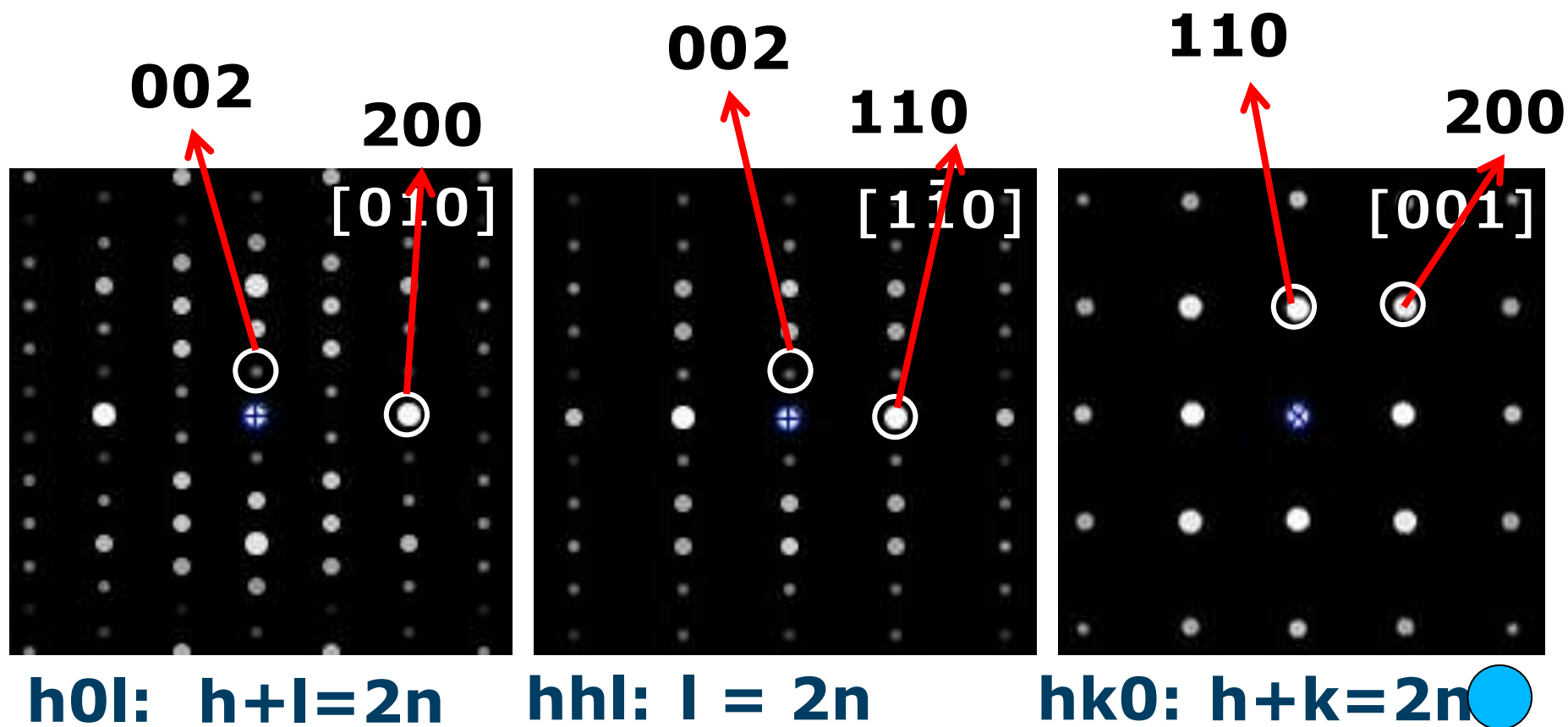
$hhl: l = 2n$

$hk0: h+k=2n$ ●

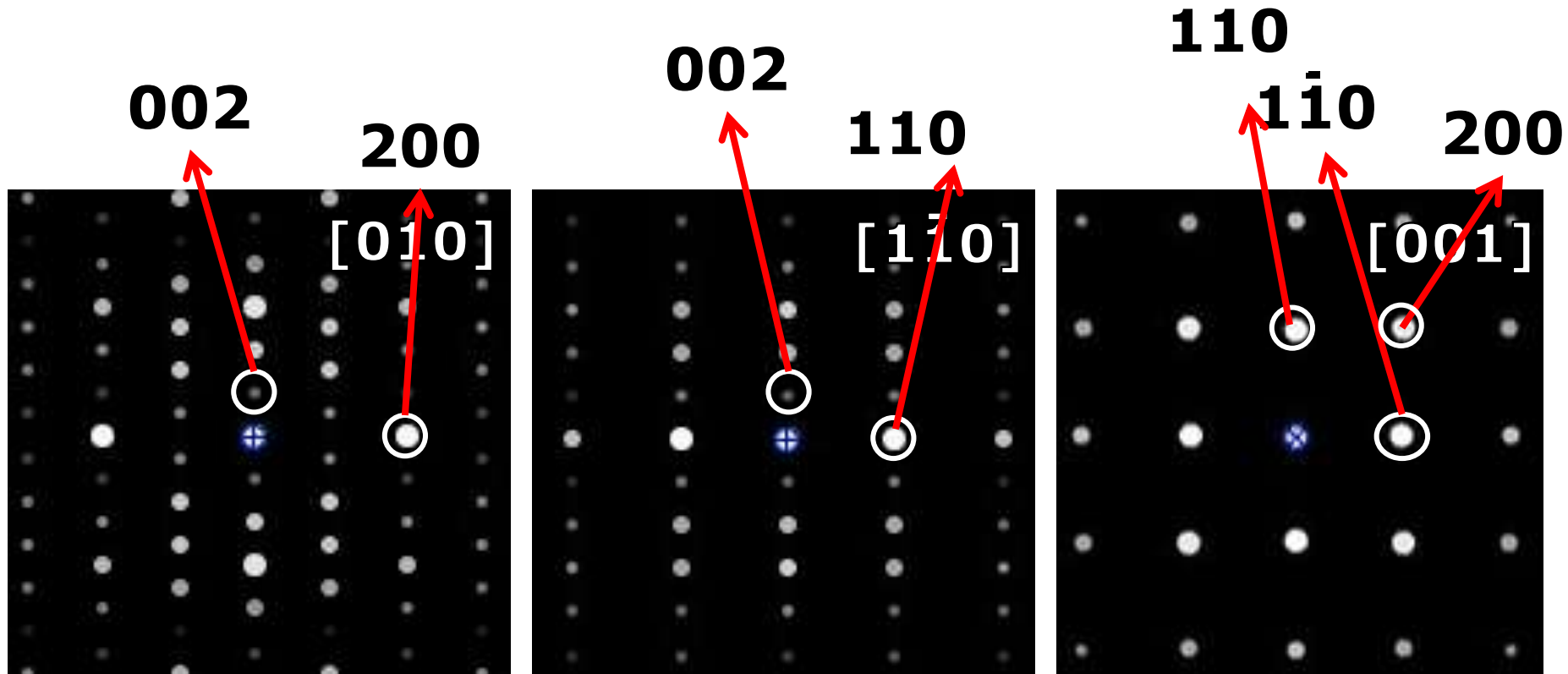
$h = 2n$ ●

$k = 2n$ ●

Exercise 1-2: IMS: determine the reflection condition $hk0$



Solved reflection conditions



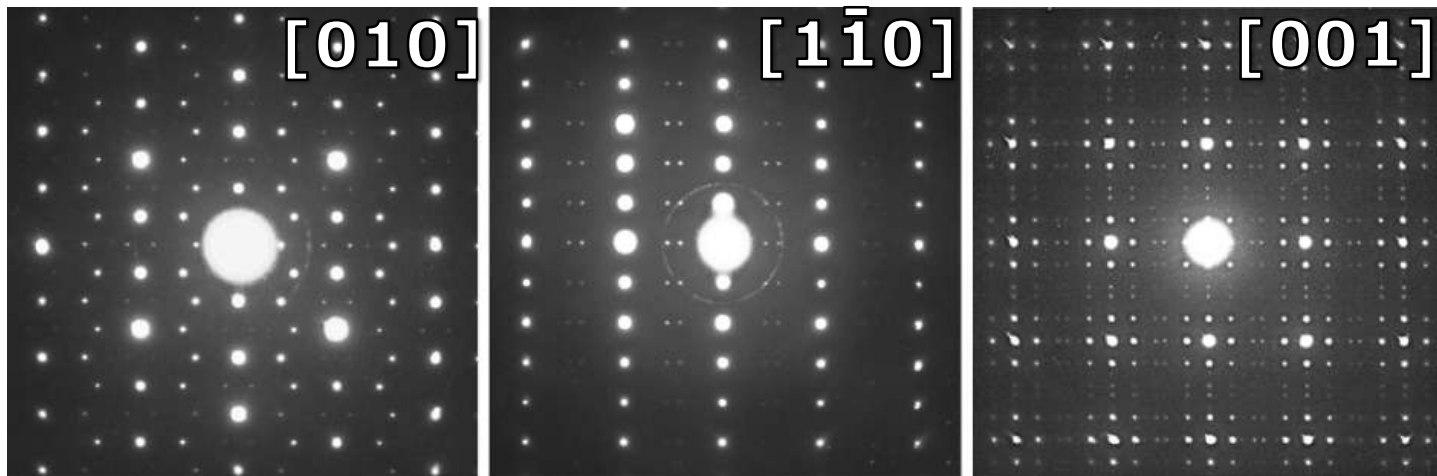
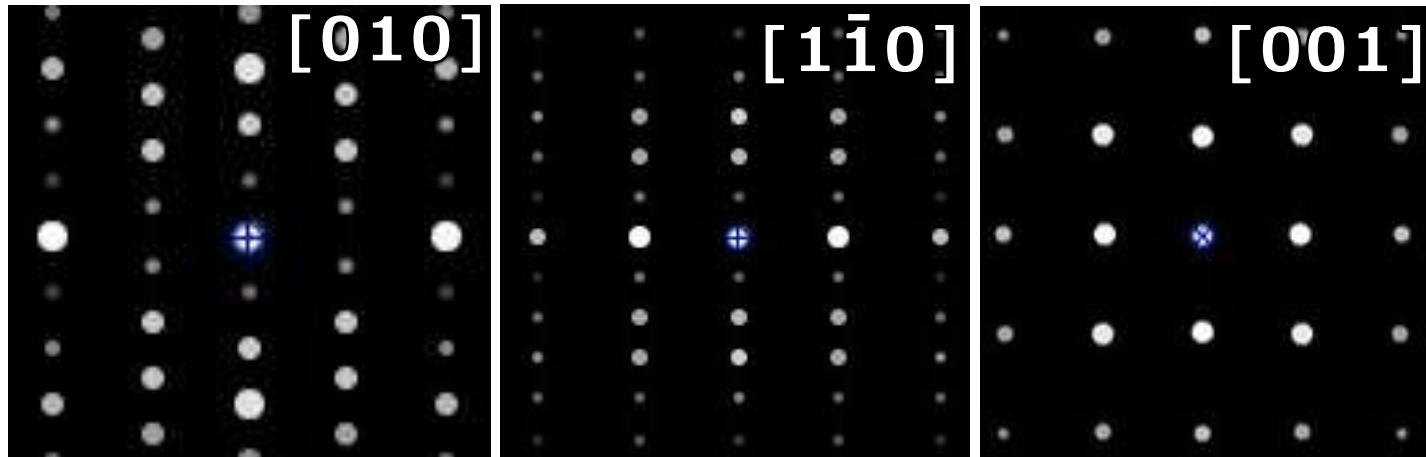
$$h0l:h+l=2n$$

$$hhl:l=2n$$

$$hk0:h+k=2n$$

Also (from rest of the zones) $hkl: h+k+l=2n$.

Incommensurate vs. basic cell

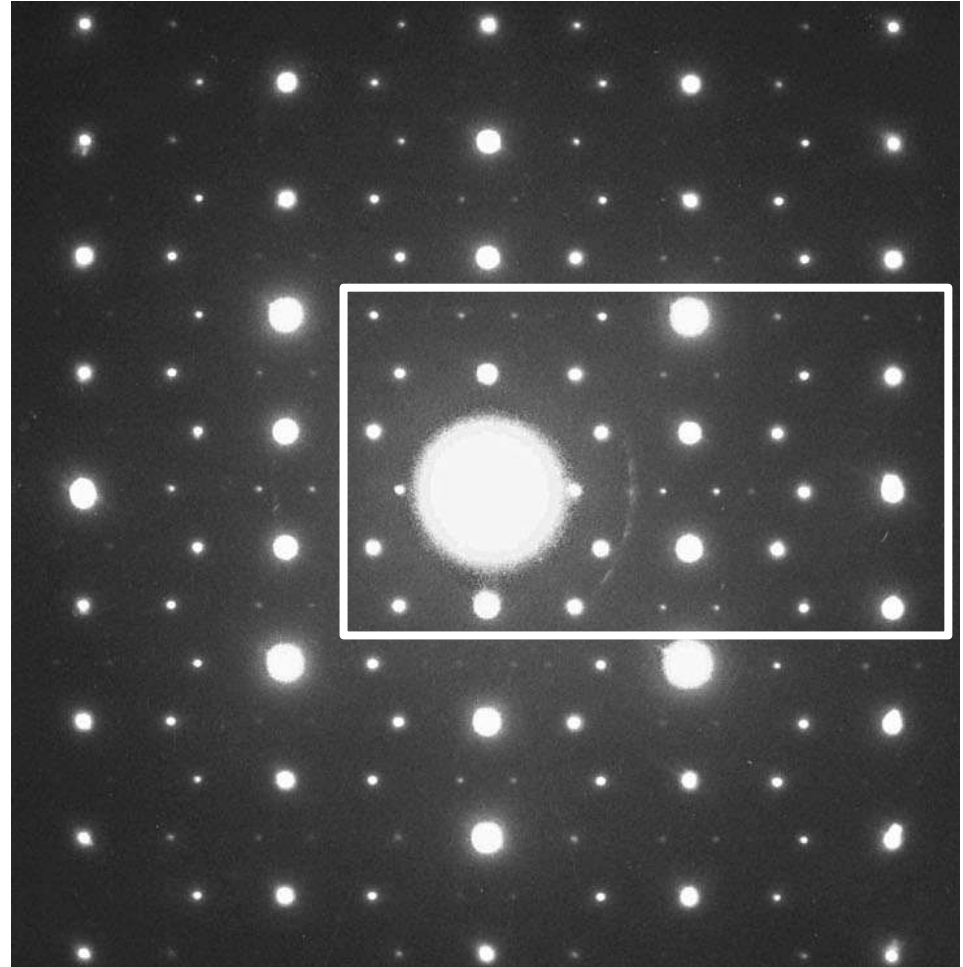


Hadermann et al., Journal of Materials Chemistry, 17, 22, 2007, 2344-2350

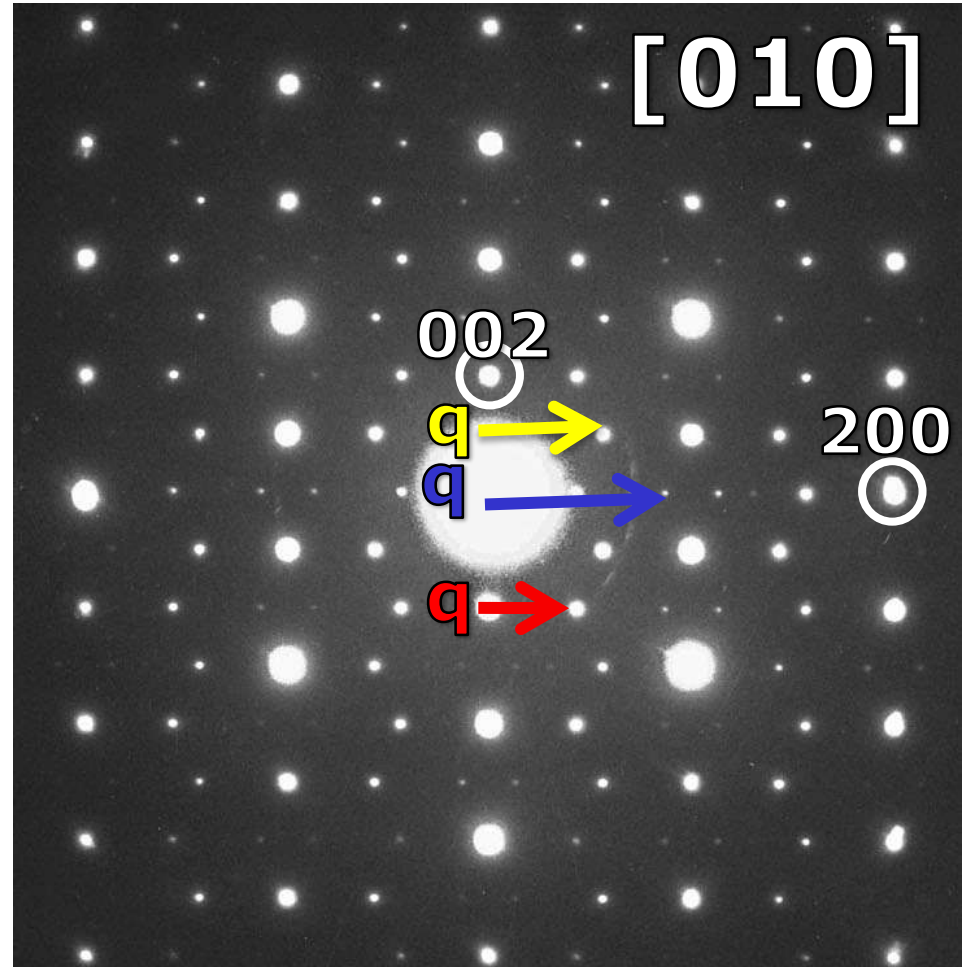


Exercise 1-3: index the IMS

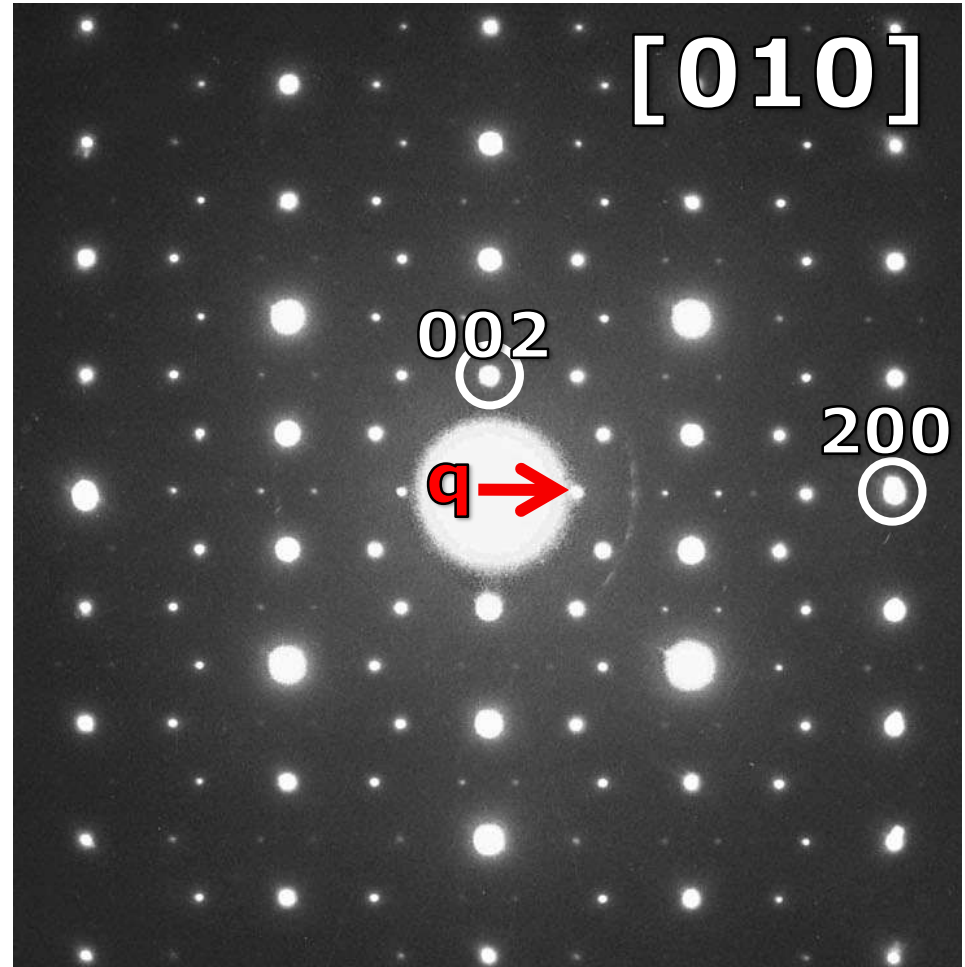
Identify and index the subcell reflections.



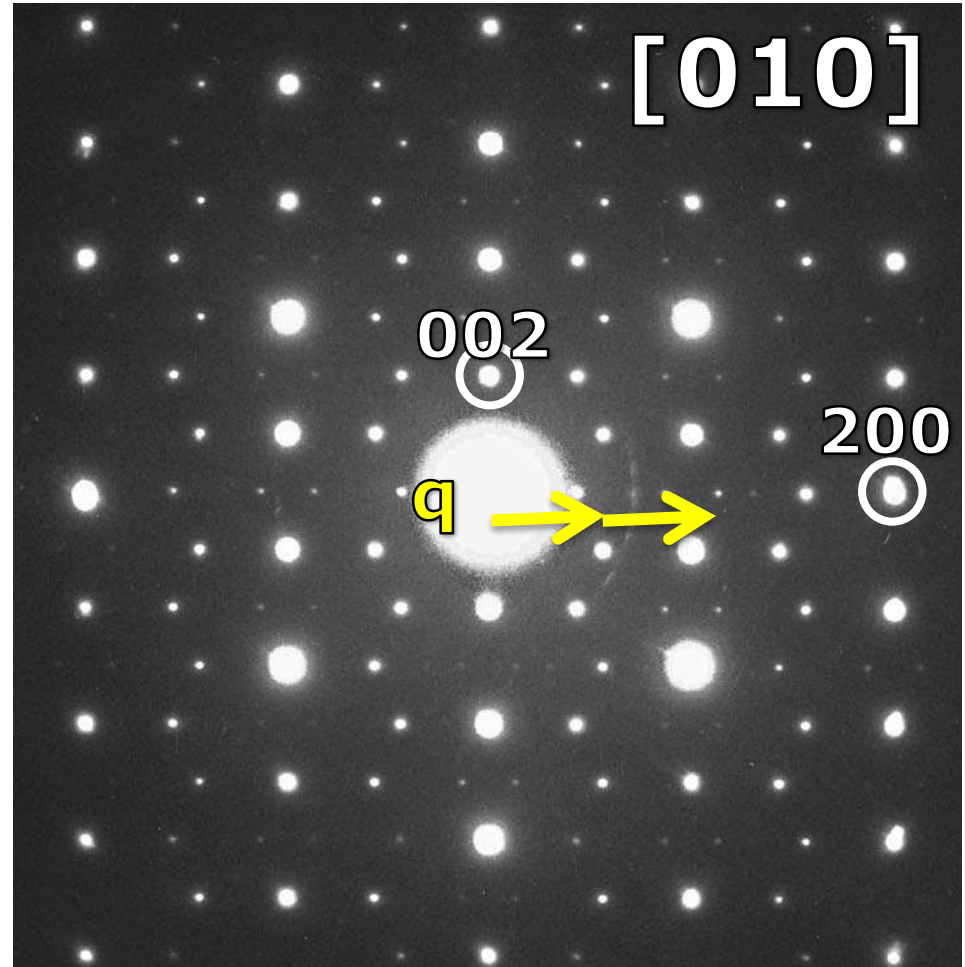
Propose a modulation vector.



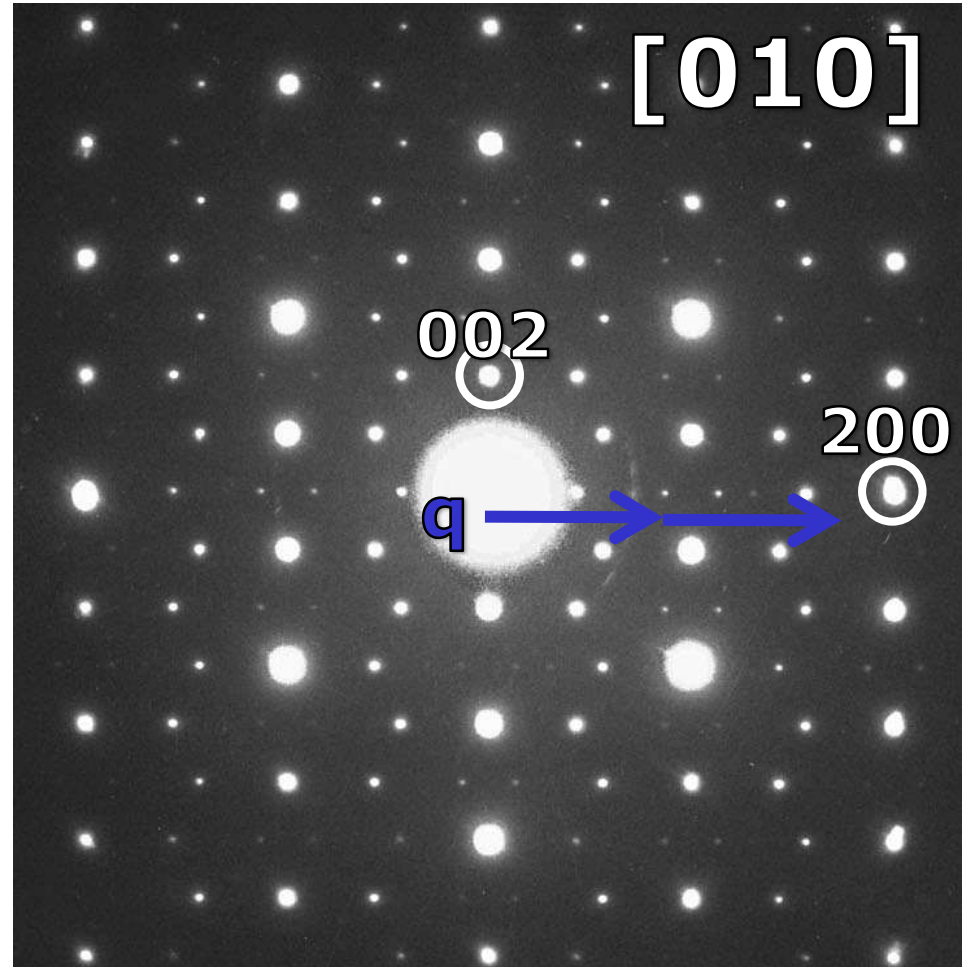
- $q = \alpha a^*$
- $\alpha < 0.5$



- $q = \alpha a^*$
- $\alpha < 0.5$

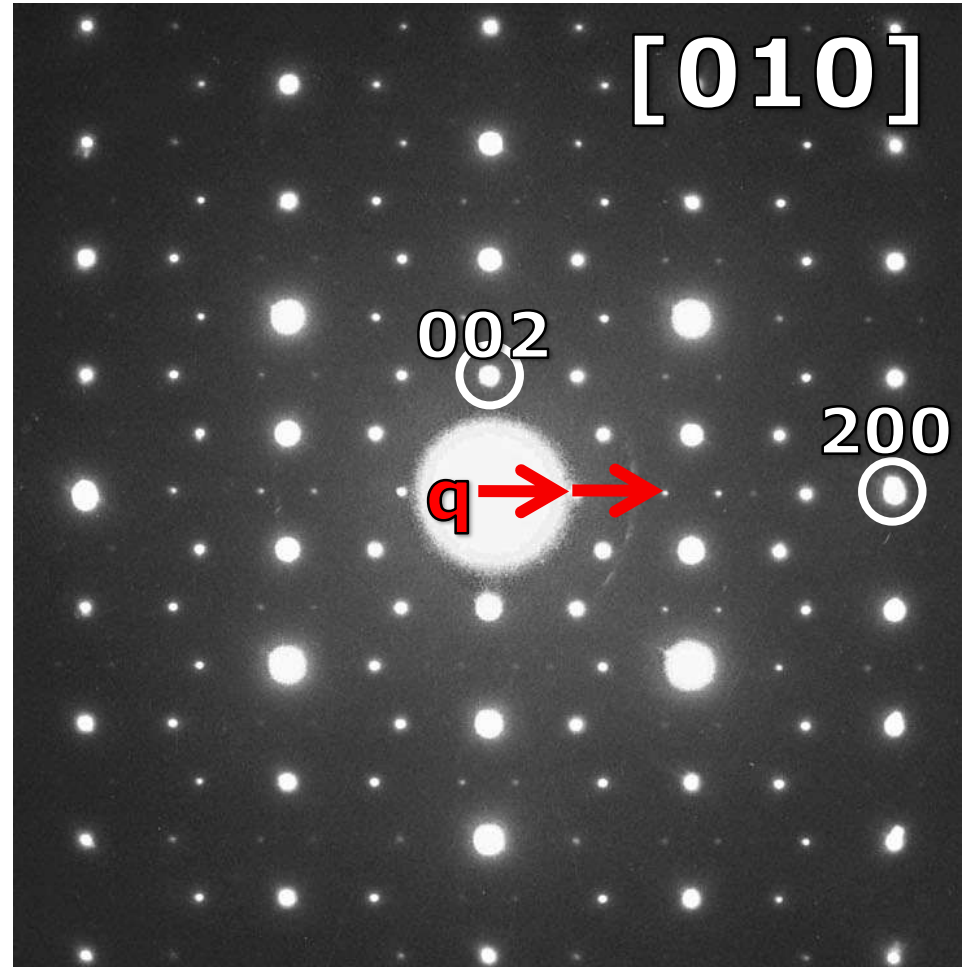


- $q = \alpha a^*$
- $\alpha < 0.5$



Possible solution

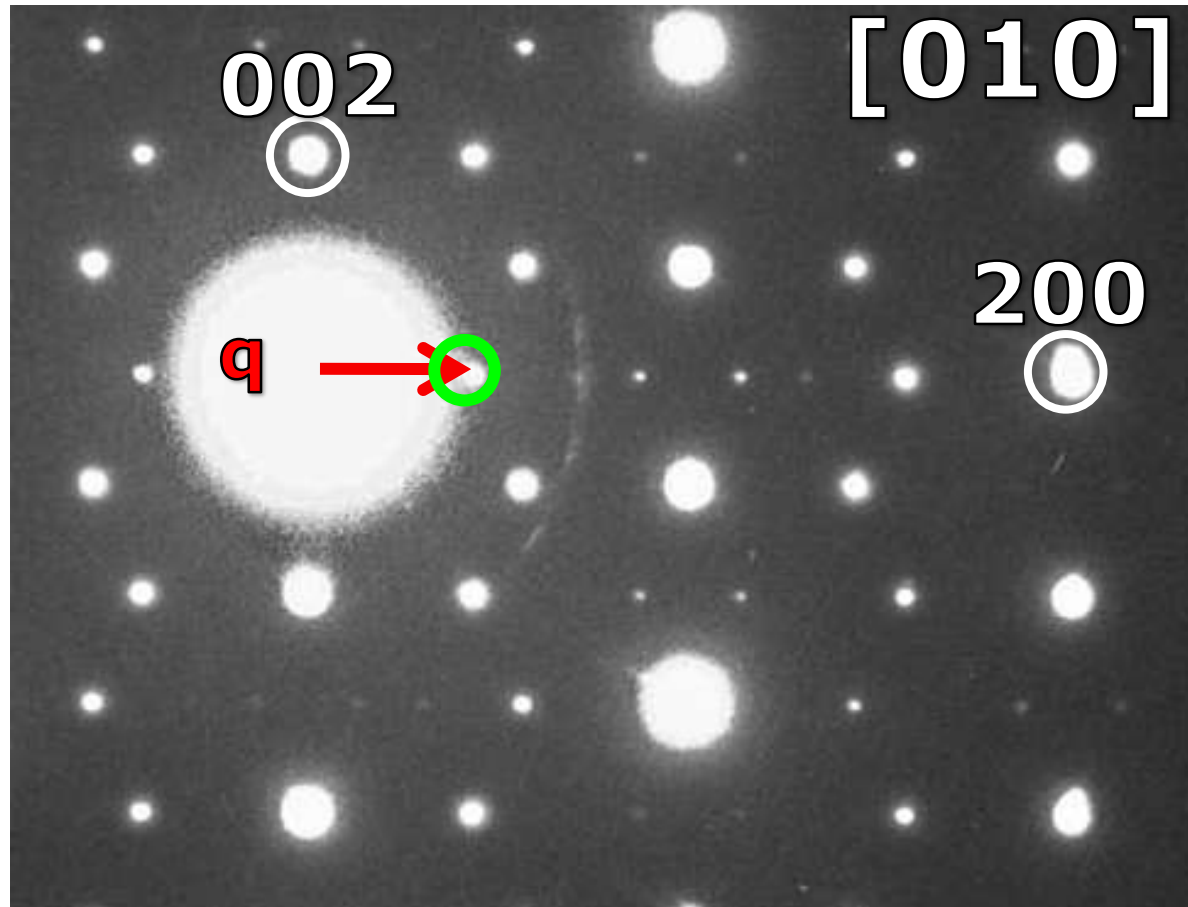
- $q = \alpha a^*$
- $\alpha < 0.5$



Index the satellite indicated in green



- 001
- 0001
- 100

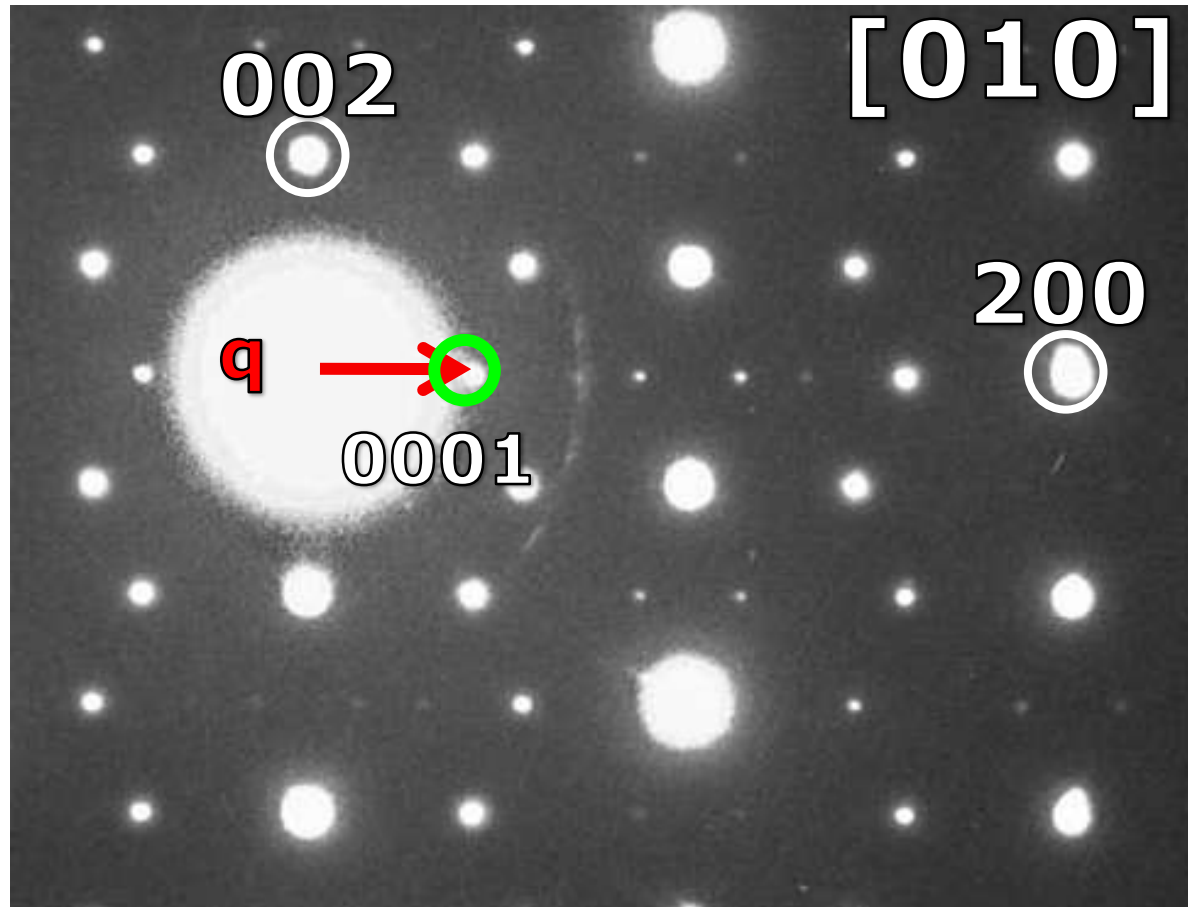


LaSrCuO_{3.52}

Index the satellite indicated in green



● 0001

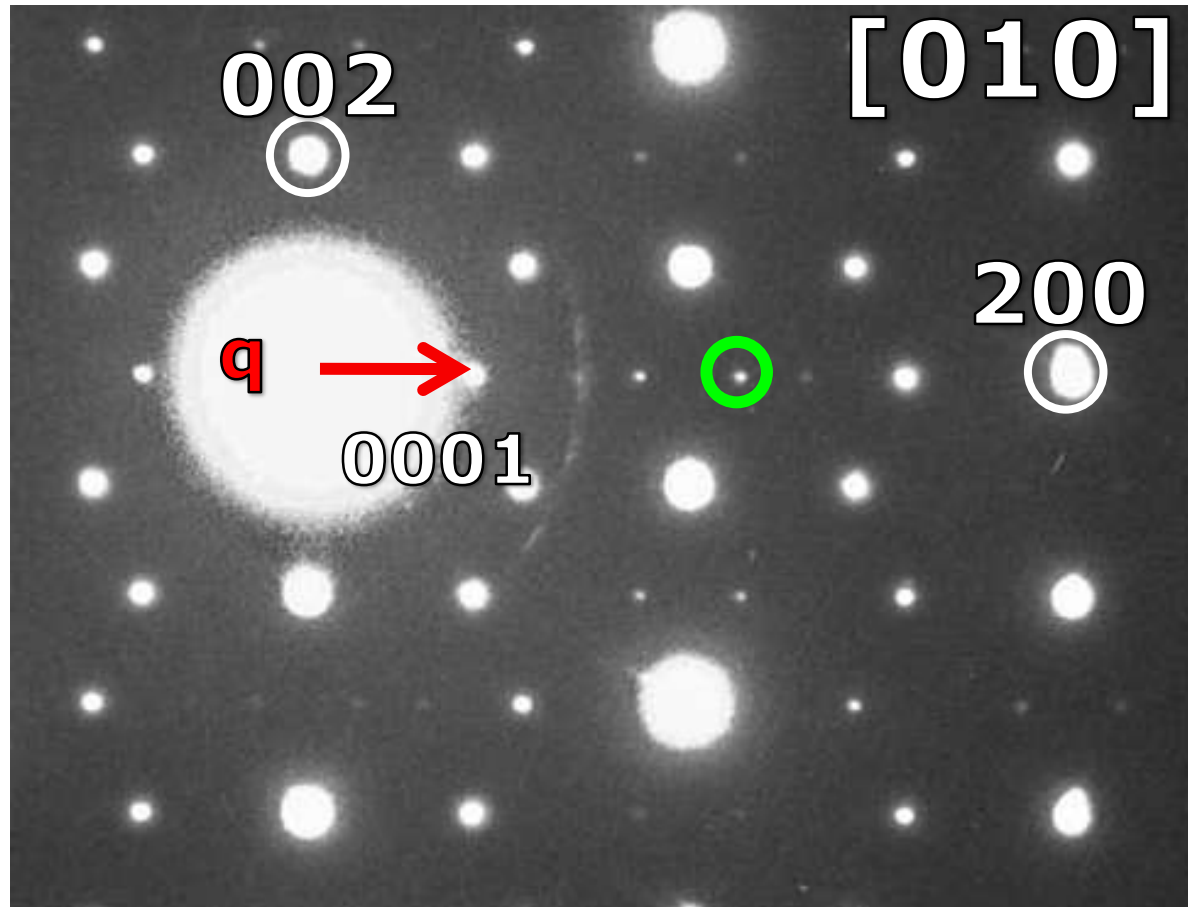


LaSrCuO_{3.52}

Index the next indicated satellite



- 0002
- $200\bar{1}$
- $200\bar{2}$

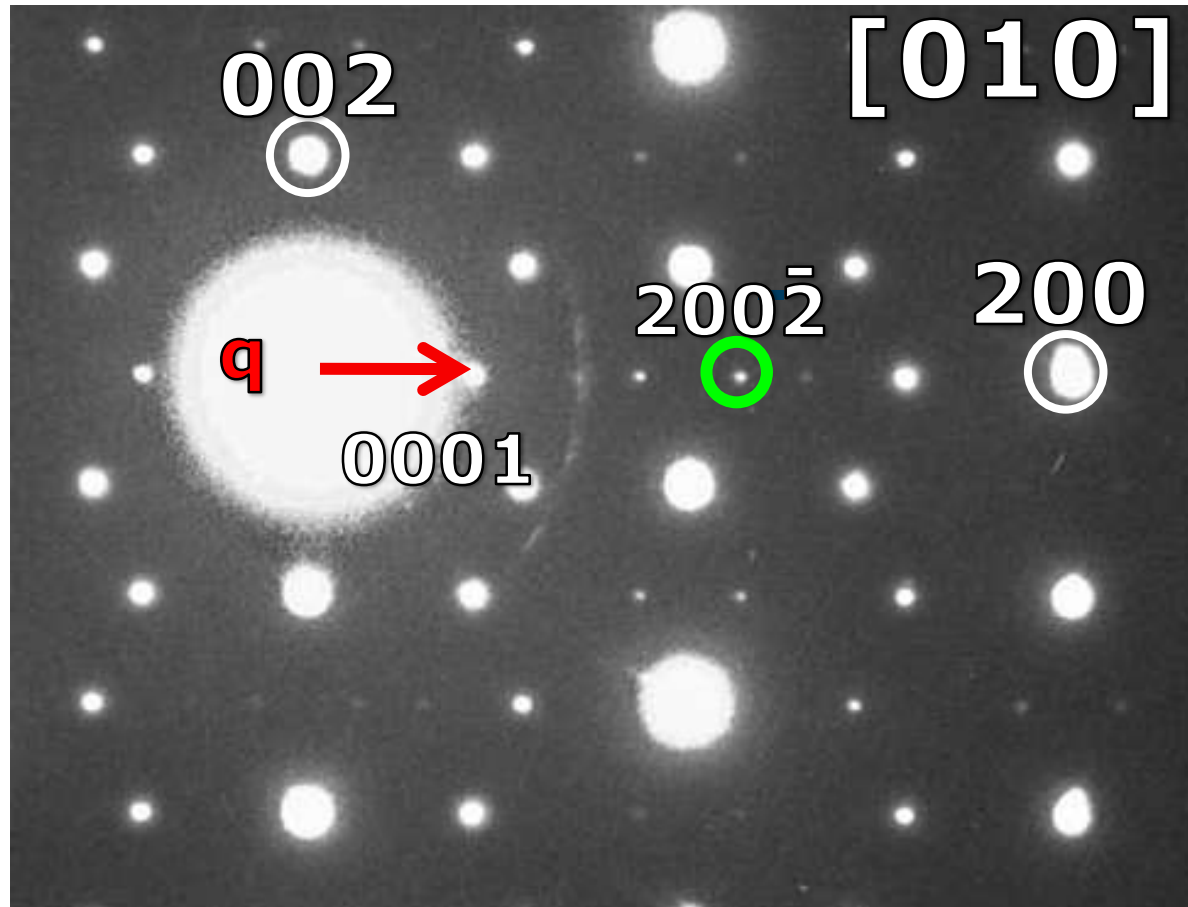


LaSrCuO_{3.52}

Index the next indicated satellite

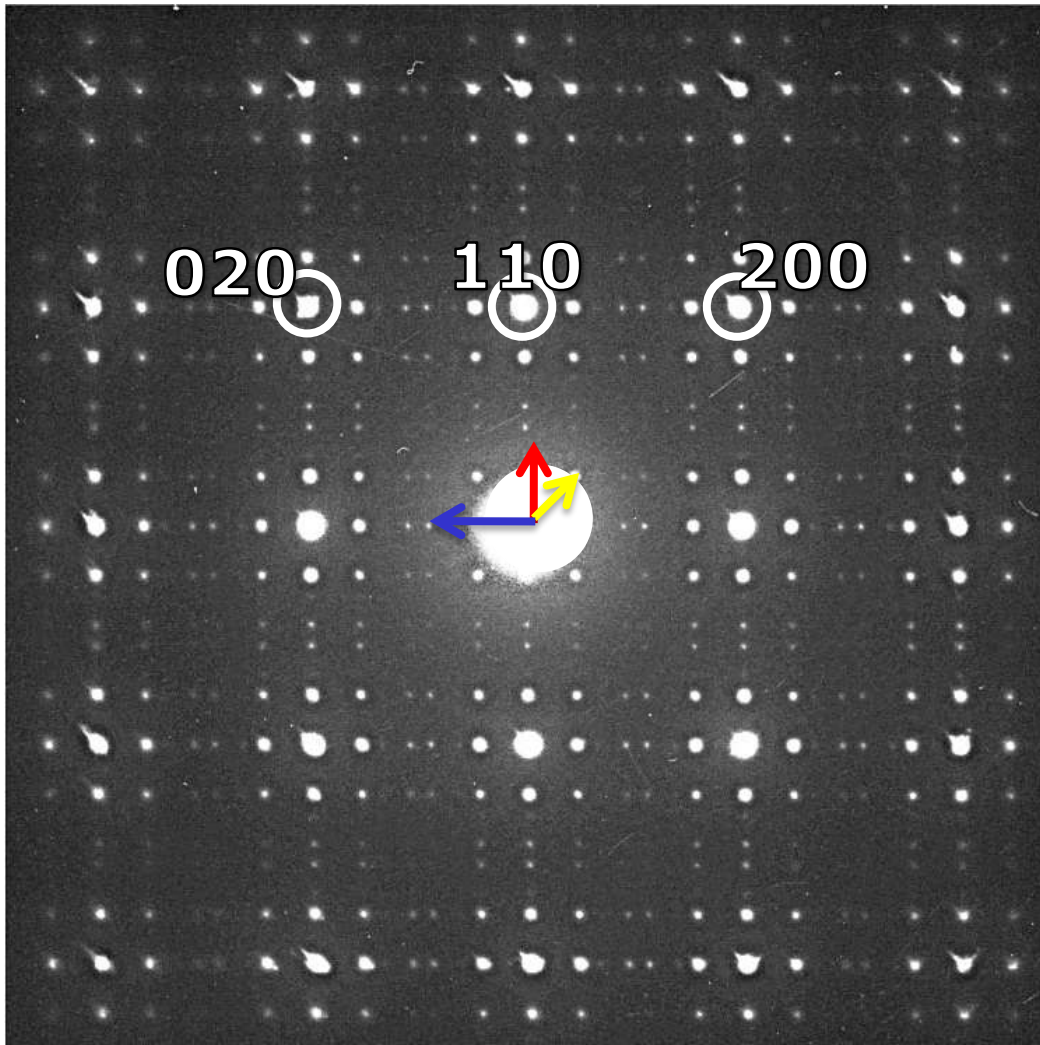


● $200\bar{2}$



LaSrCuO_{3.52}

Exercise 1-4: IMS with more complete data

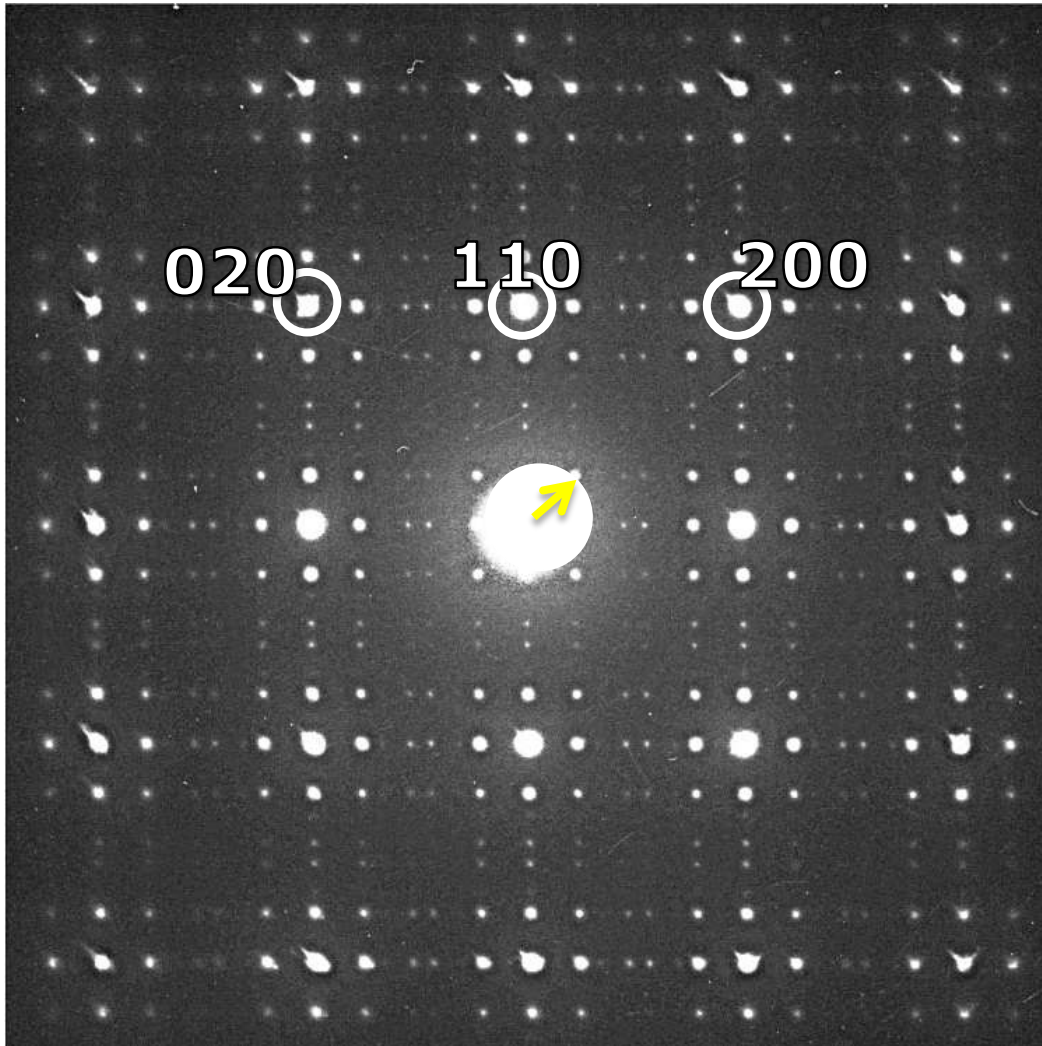


- Which of the indicated vectors corresponds to the modulation vector chosen on the previous slides?



$\text{LaSrCuO}_{3.52}$

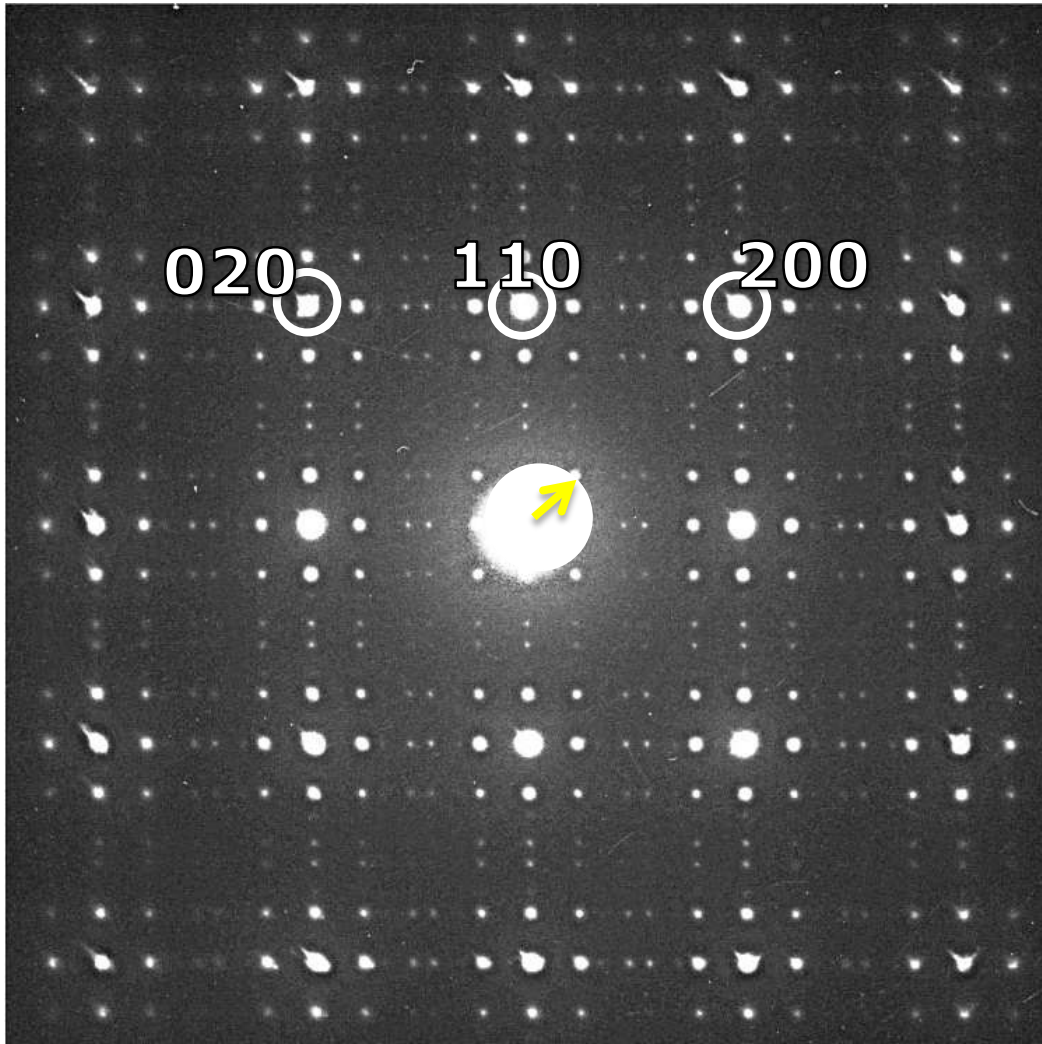
Exercise 1-4: IMS with more complete data



- Which of the indicated vectors corresponds to the modulation vector chosen on the previous slides?



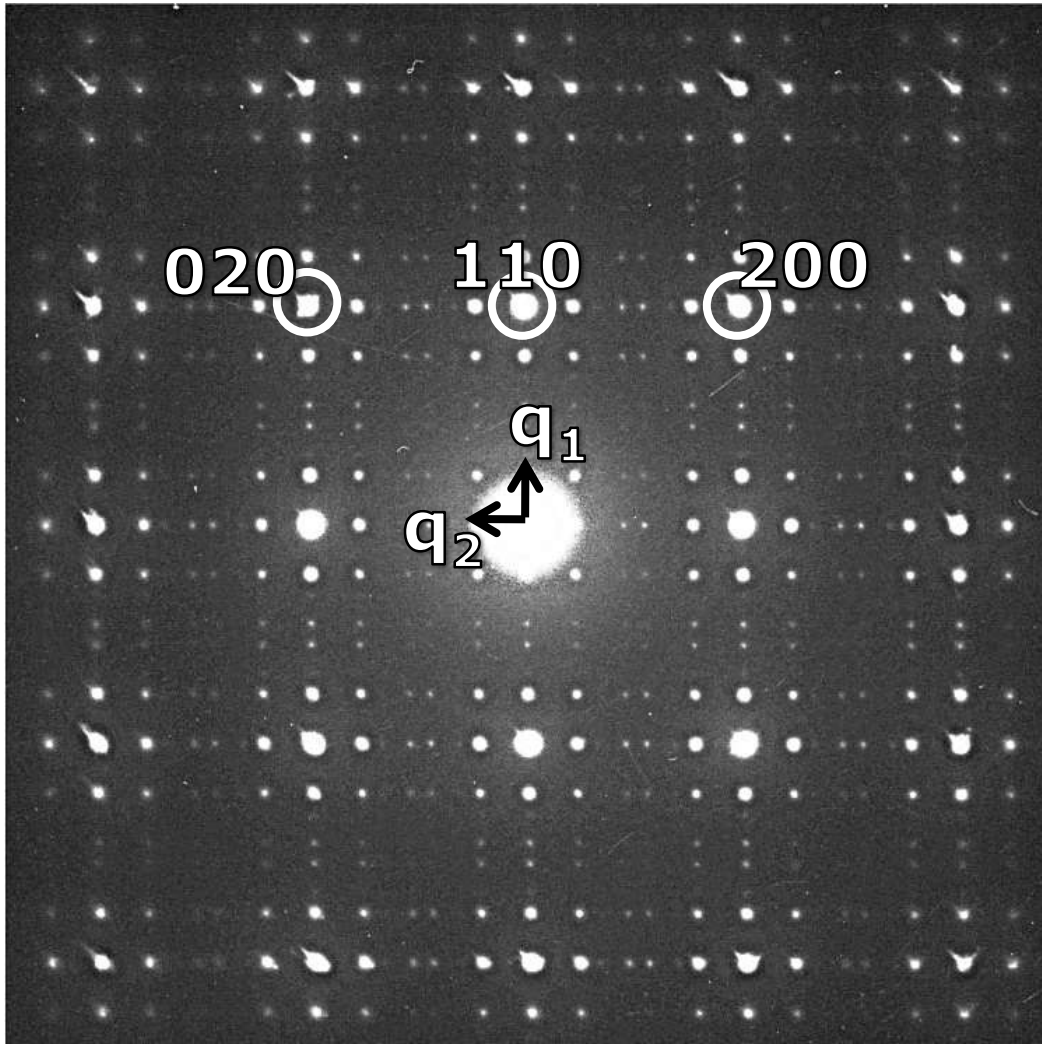
LaSrCuO_{3.52}



Is the proposed vector still valid?

yes

no



Indicate the correct modulation vector(s) on the central white area.

You need

q_1

q_2

q_1 and q_2

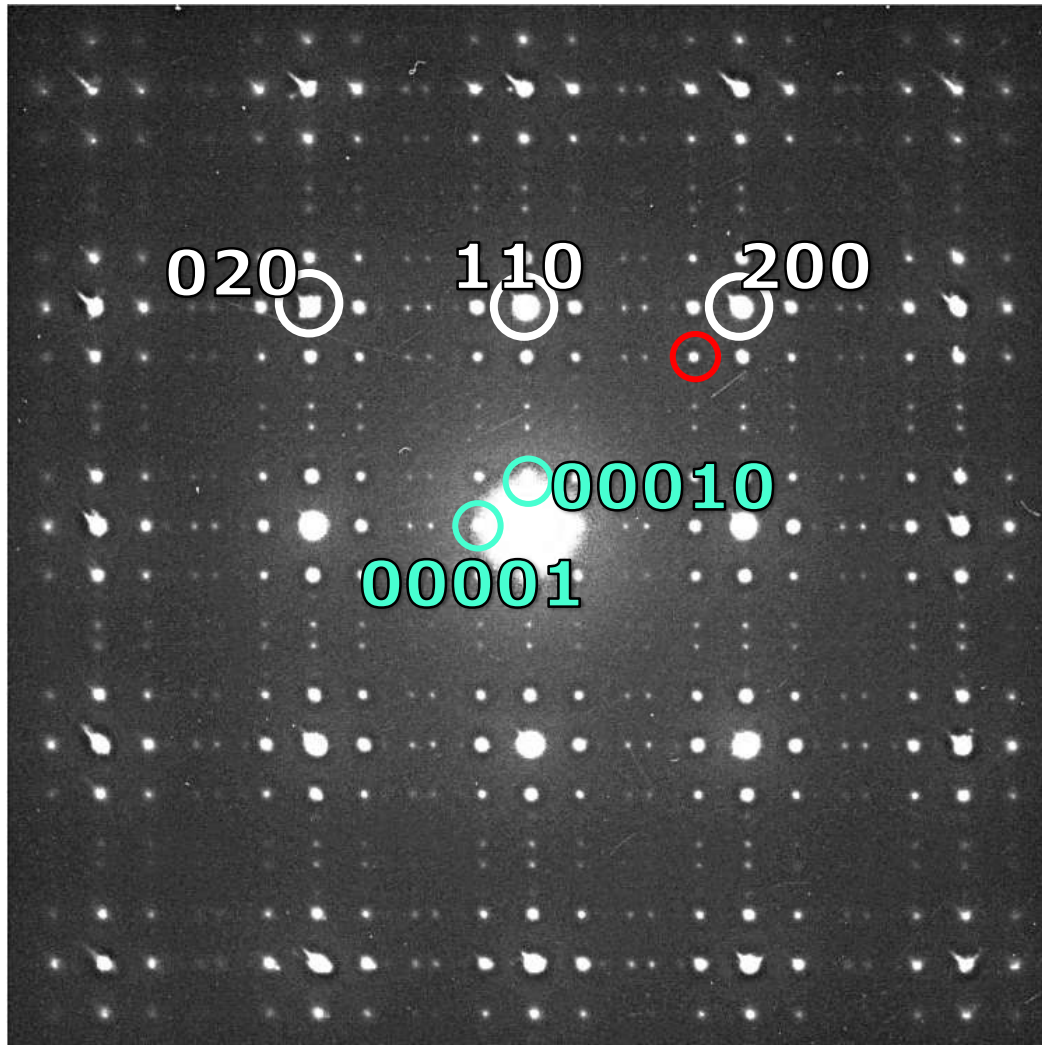
$$q_1 = \alpha a^* + \beta b^*$$

$$q_2 = -\alpha a^* + \beta b^*$$

$$\alpha = \beta < 0.25$$

hklmn

Index the reflection indicated in red

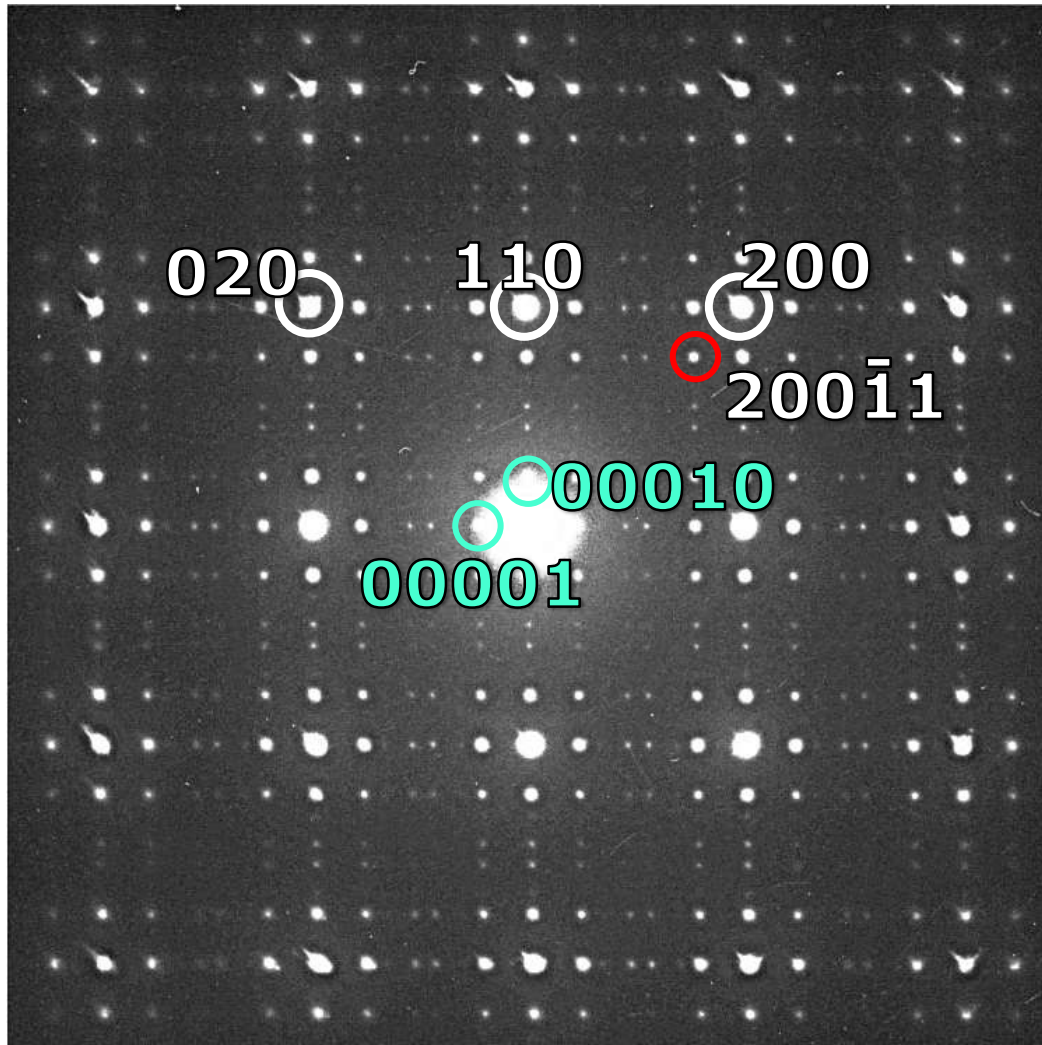


- 20011
- $200\bar{1}1$
- $2001\bar{1}$



LaSrCuO_{3.52}

Index the reflection indicated in red

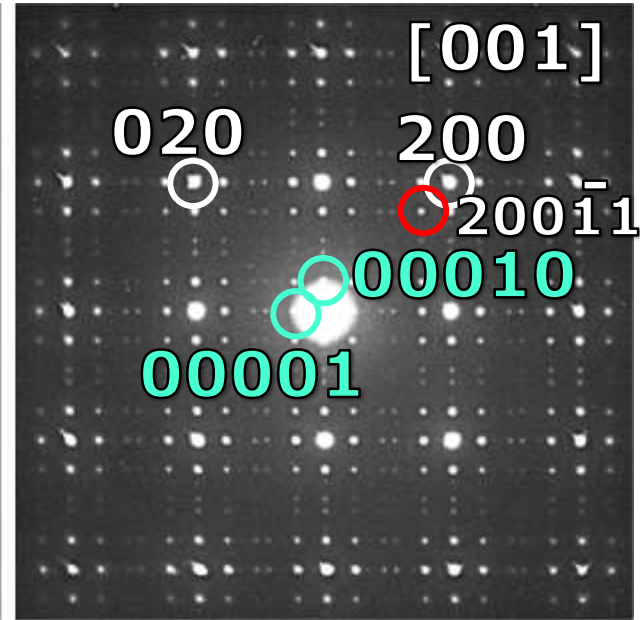
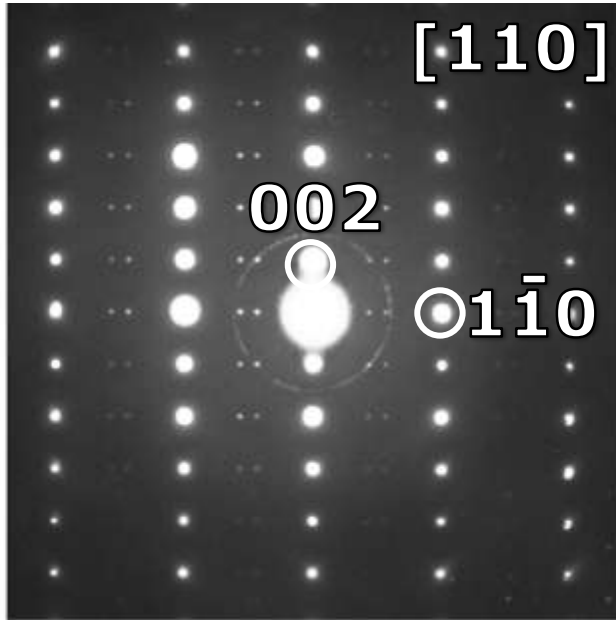
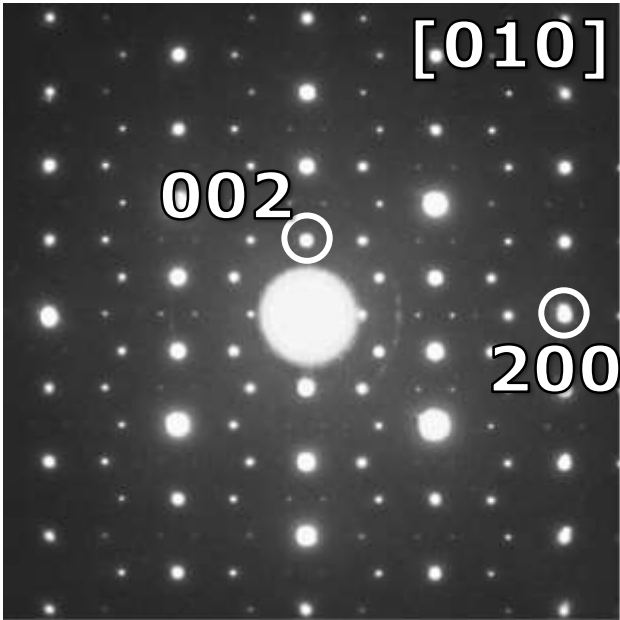


● $200\bar{1}1$

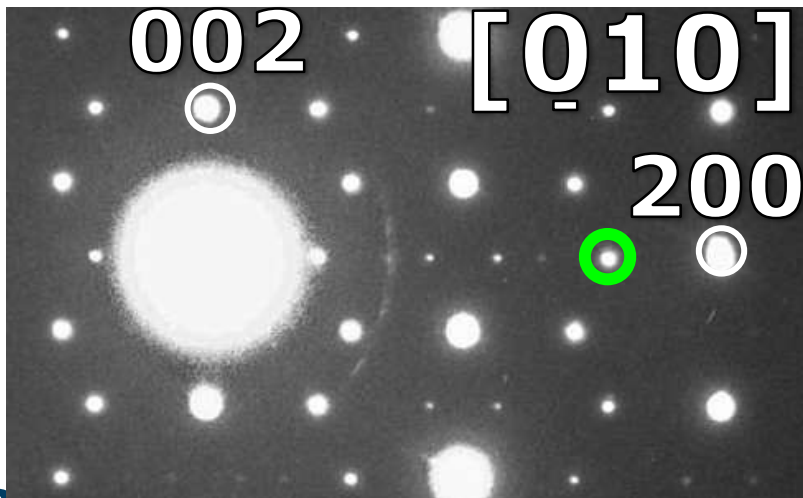
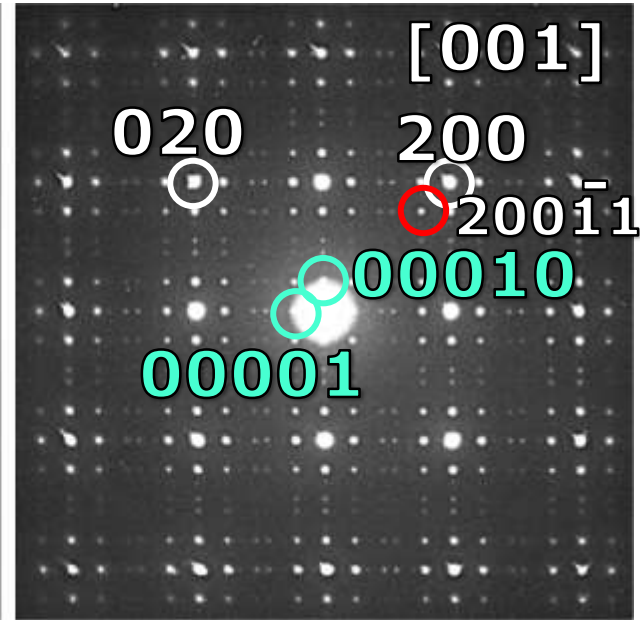
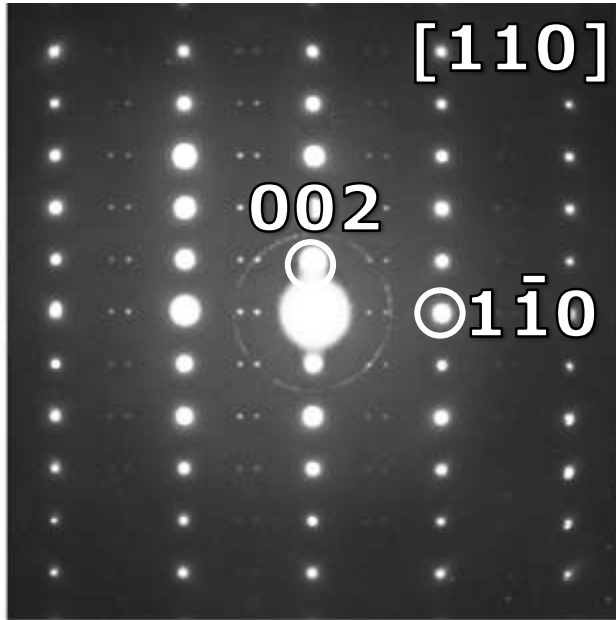
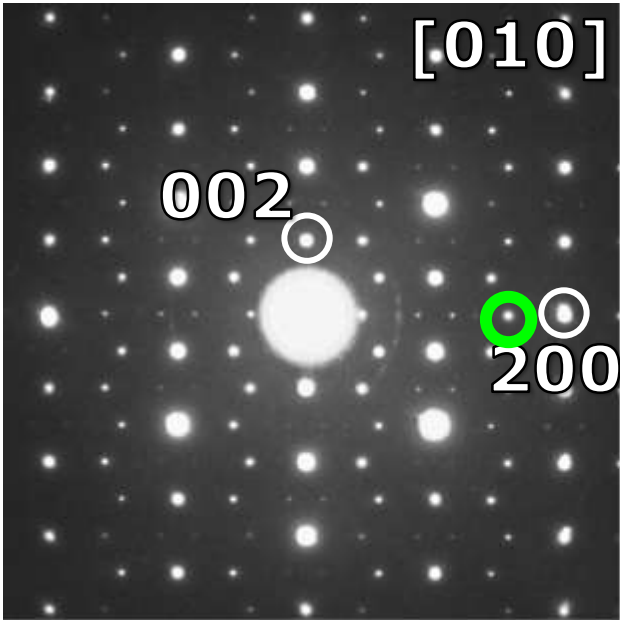


LaSrCuO_{3.52}

Index the others patterns consistently with this new choice.

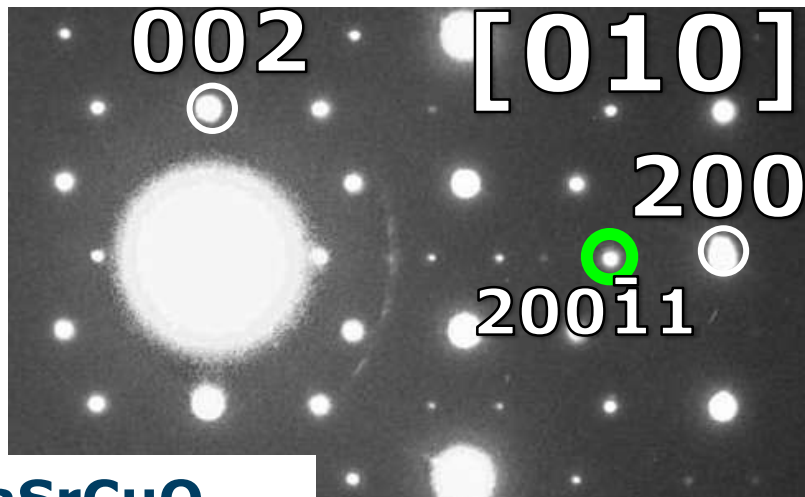
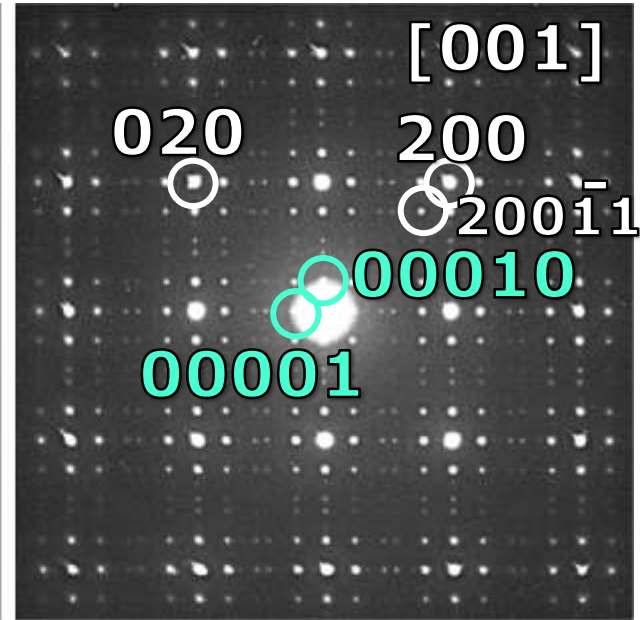
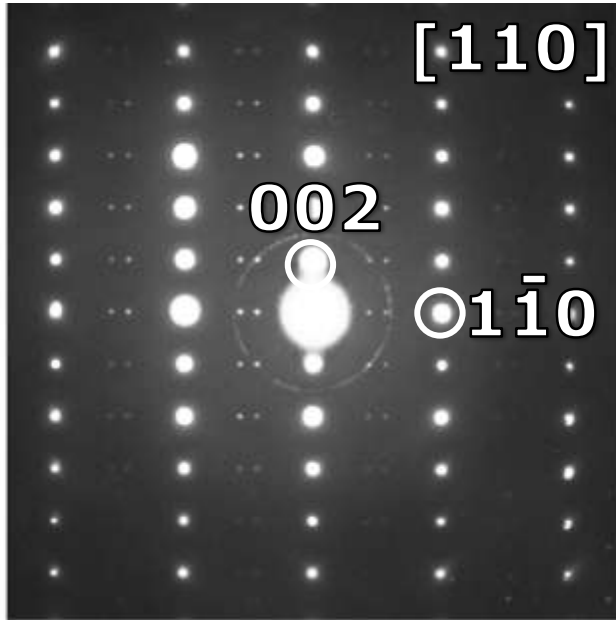
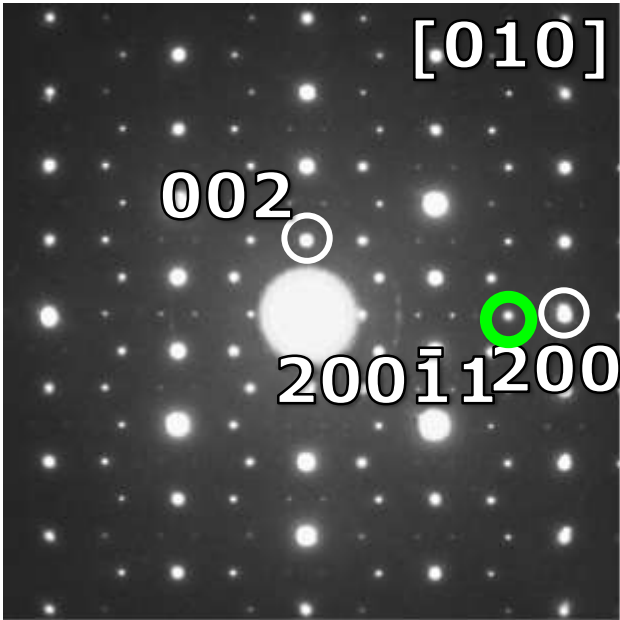


Index the reflection indicated in green



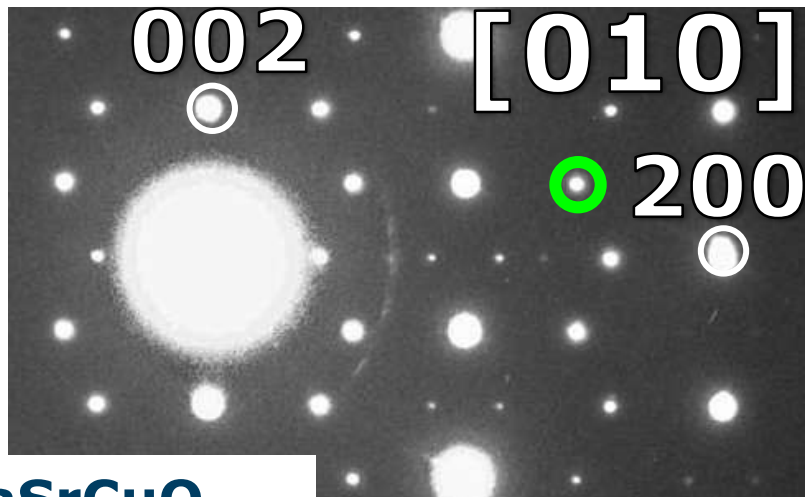
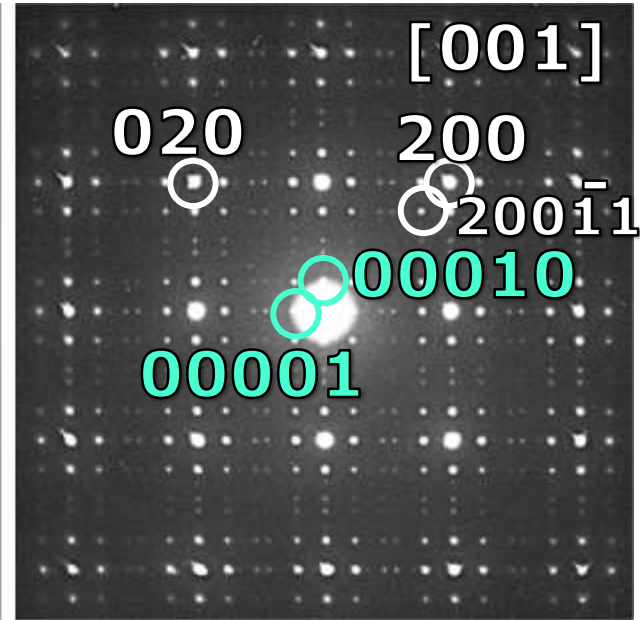
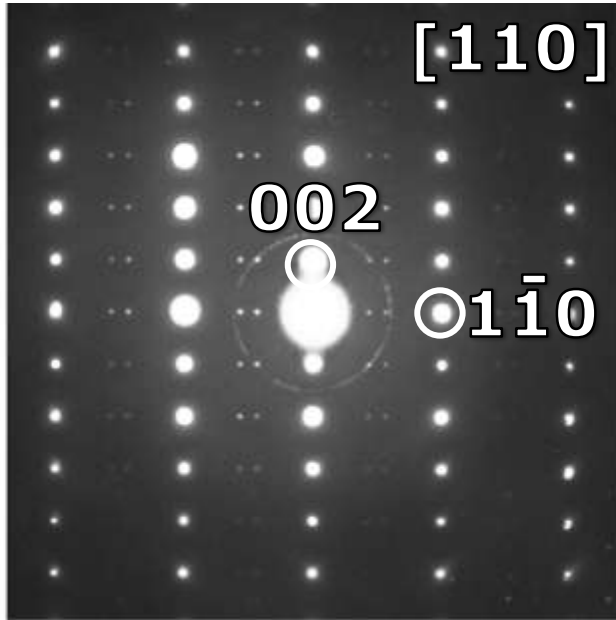
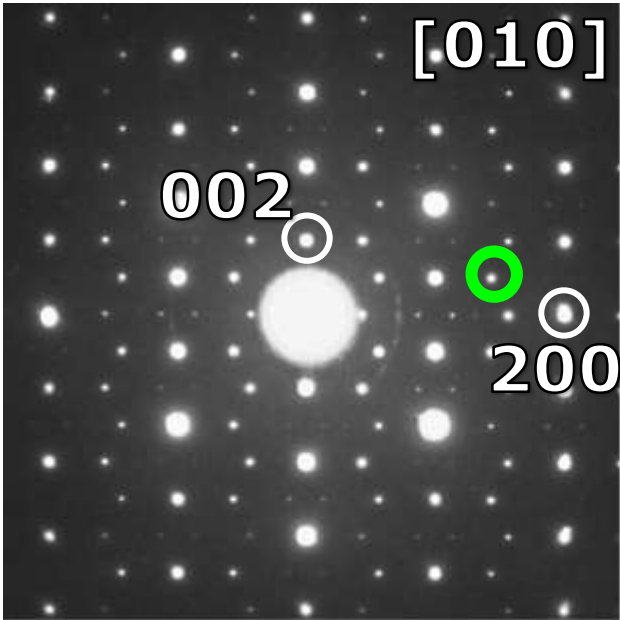
- 20011
- $200\bar{1}1$
- $2001\bar{1}$

Index the reflection indicated in green



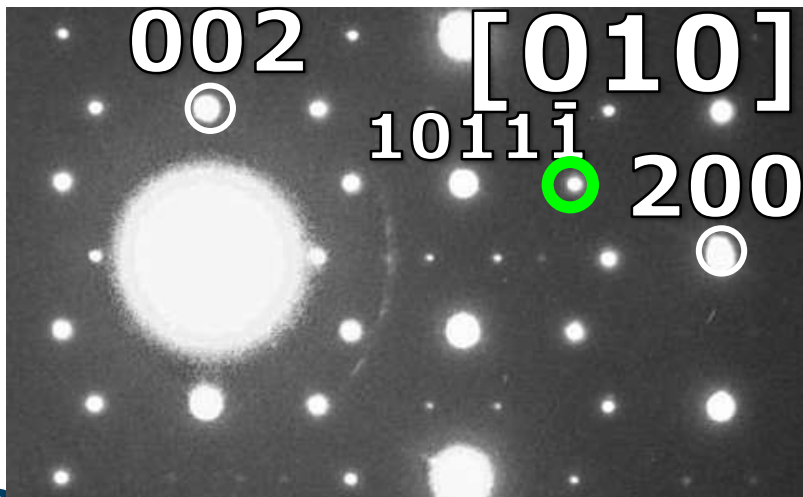
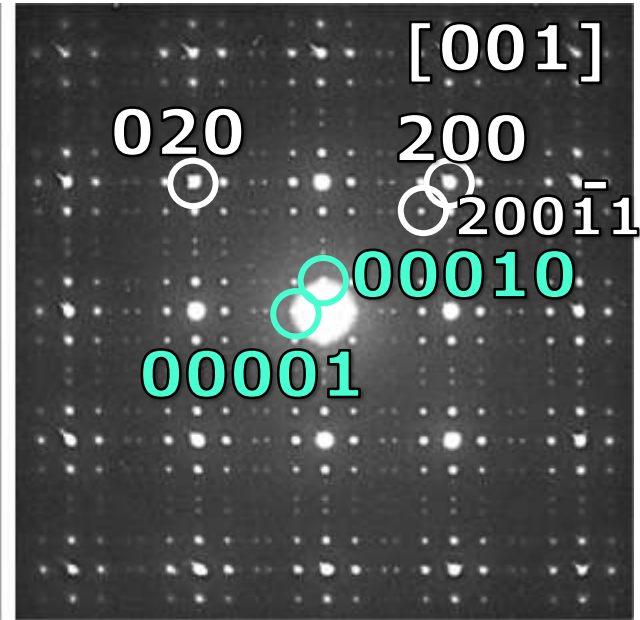
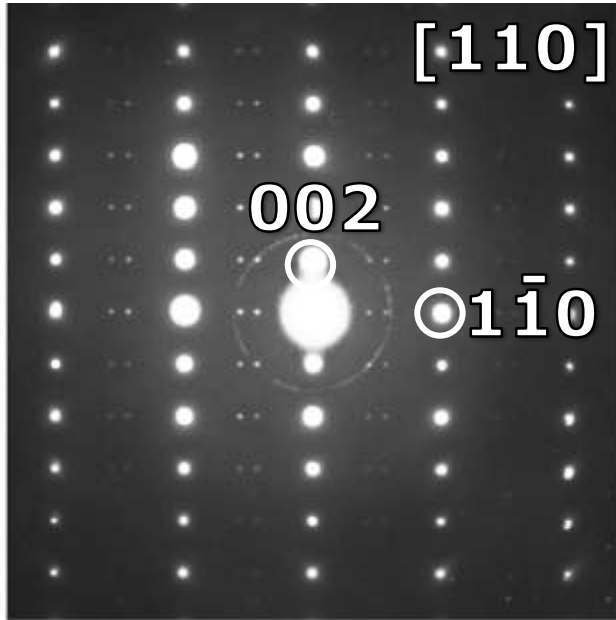
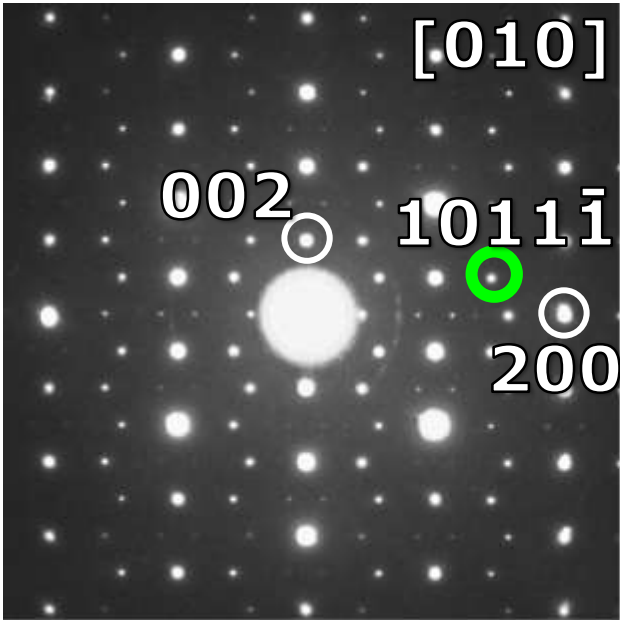
● 200 $\bar{1}1$

Index the reflection indicated in green



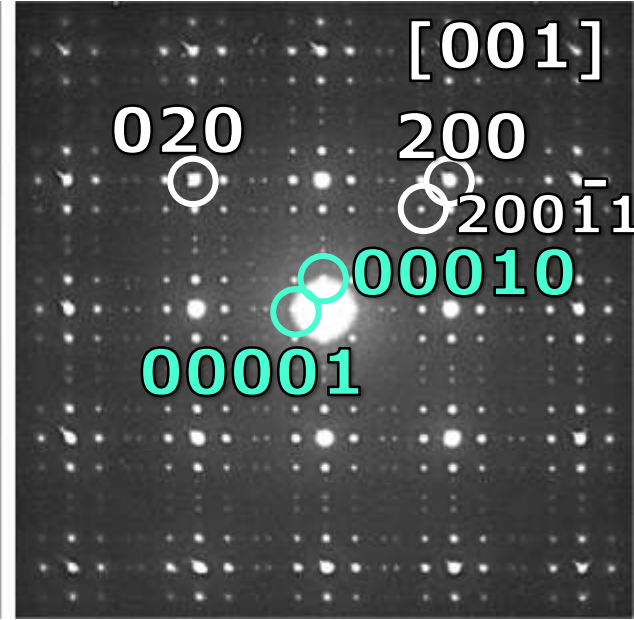
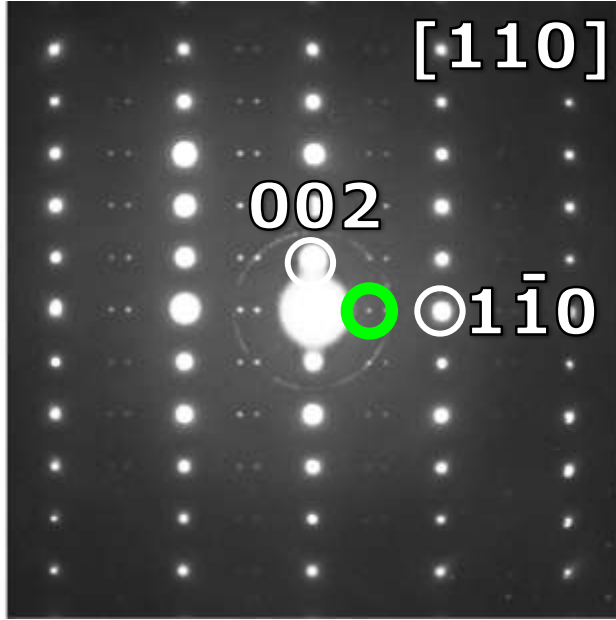
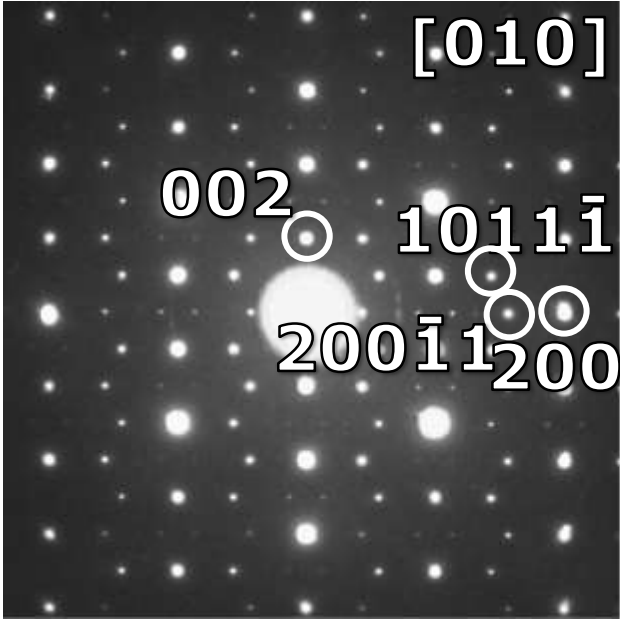
- 10110
- 10111
- 1011 $\bar{1}$

Index the reflection indicated in green



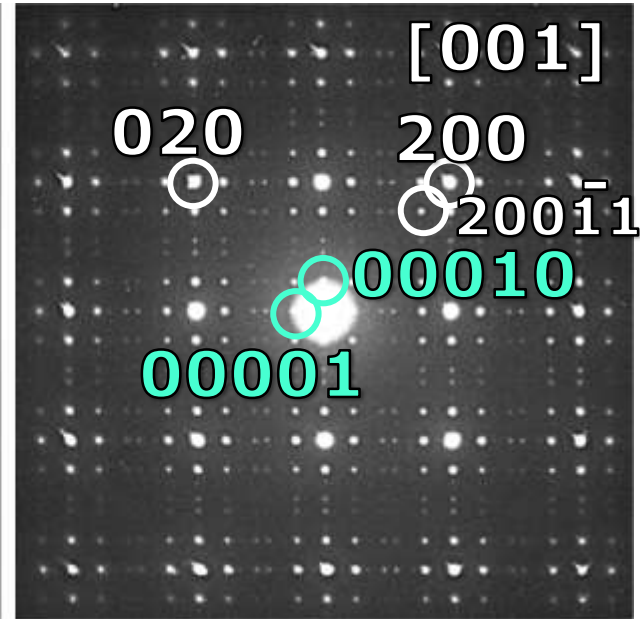
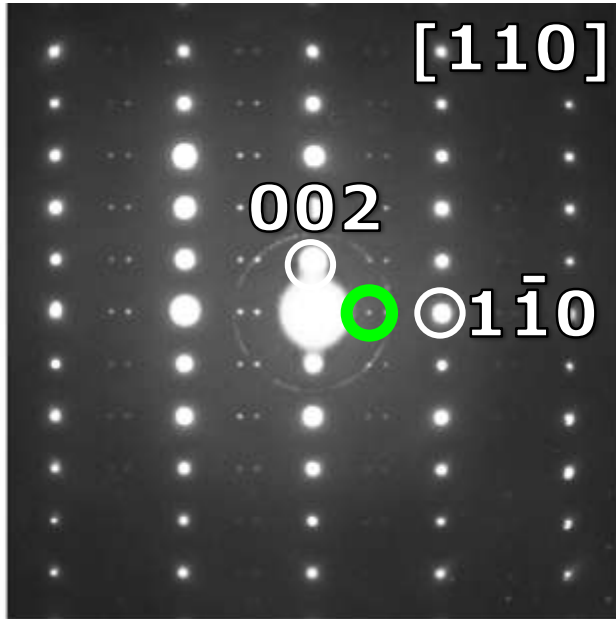
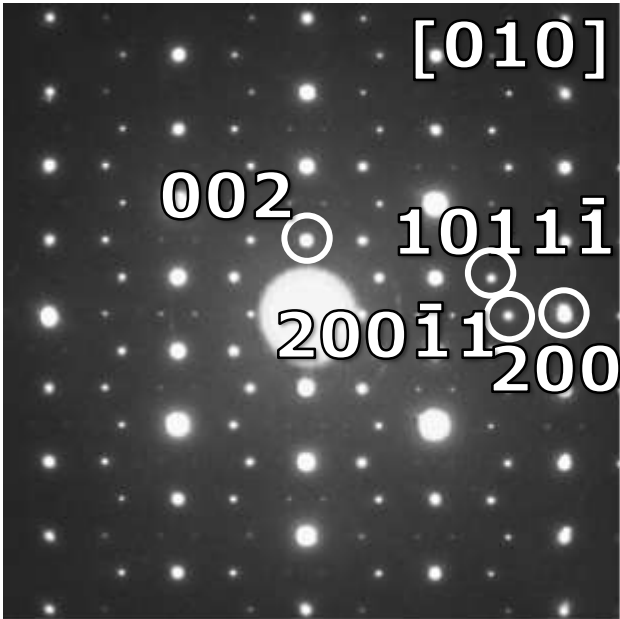
● 1011̄1̄

Index the reflection indicated in green



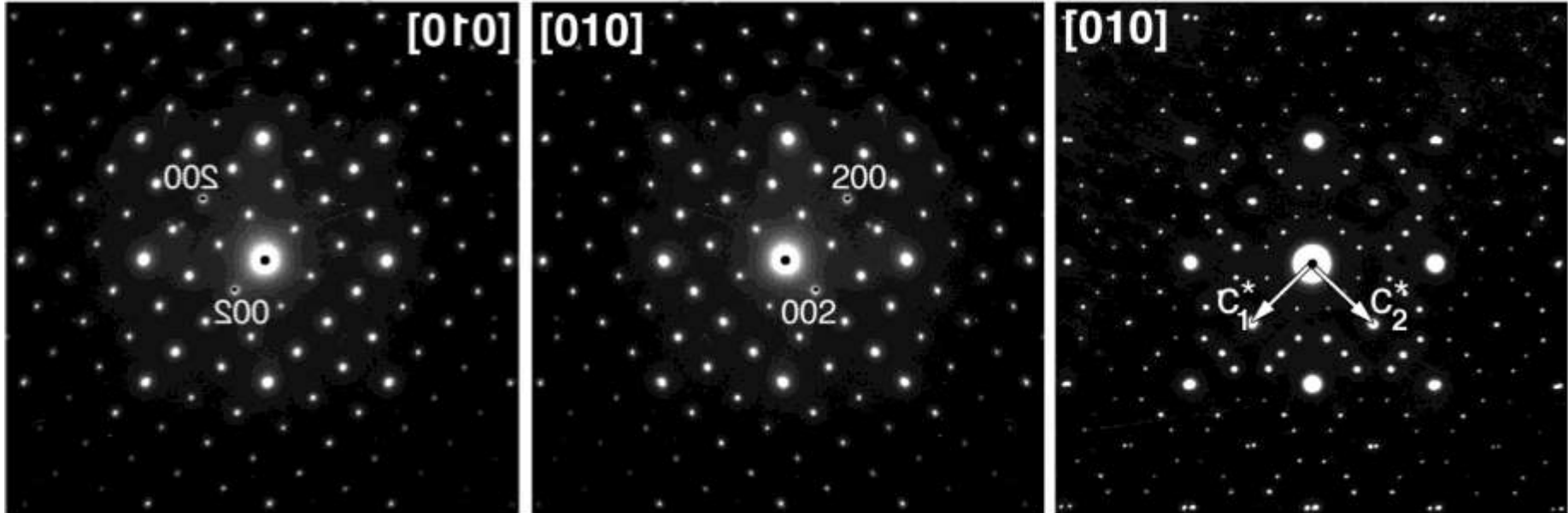
- 00002
- 00002̄
- 00001̄

Index the reflection indicated in green



● 00002̄

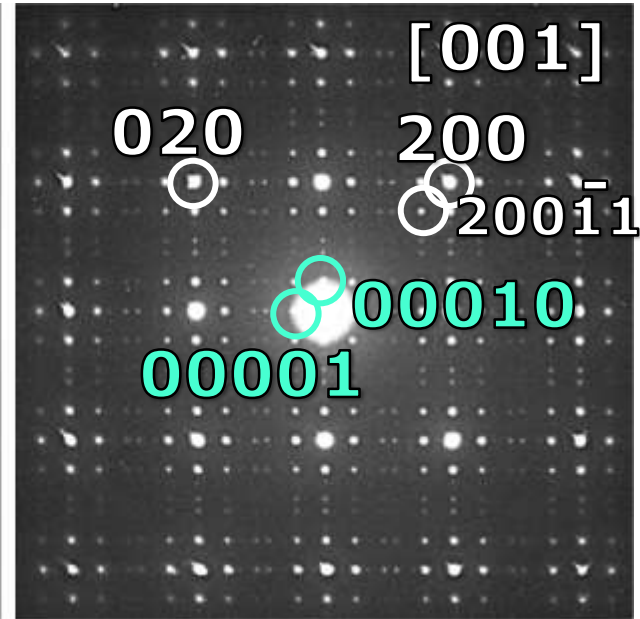
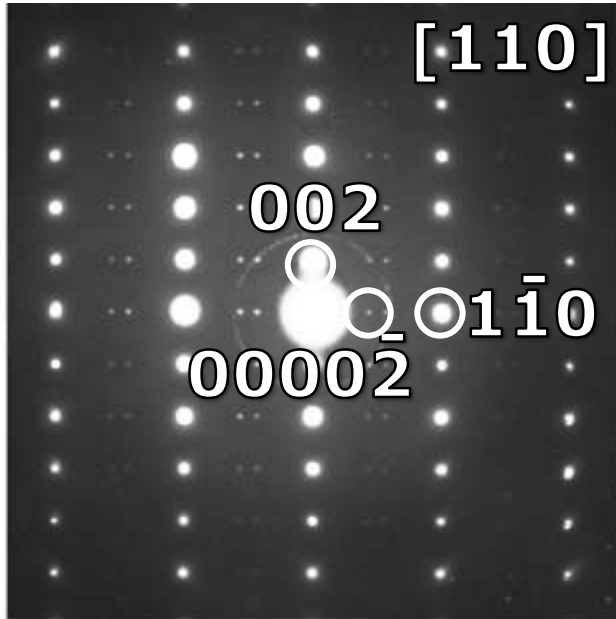
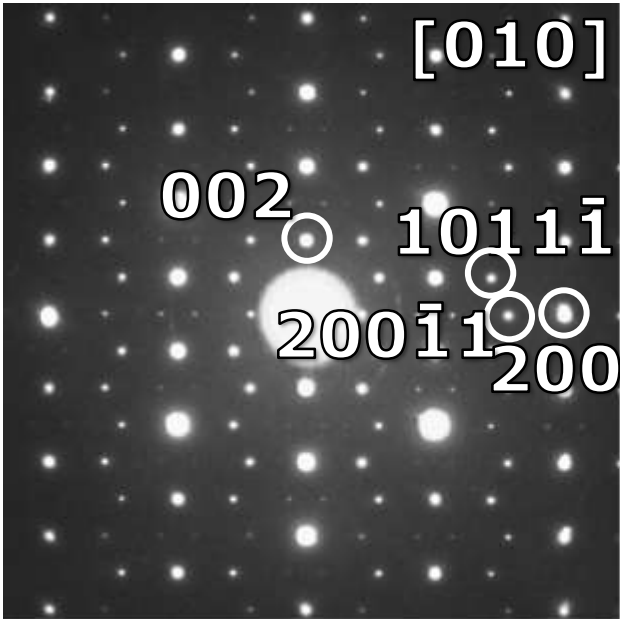
Look out for twinning!



Gillie et al., Journal of Physics and Chemistry of Solids, 65, 1 (2004) 87-93



Exercise 1-5: twinning?

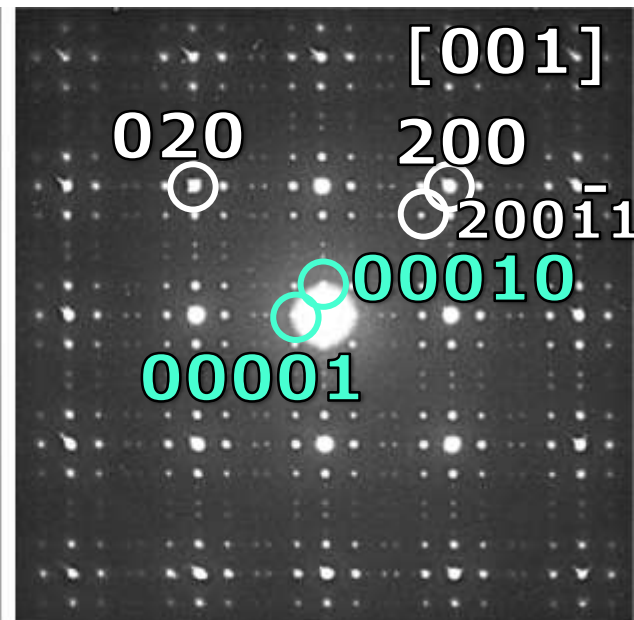
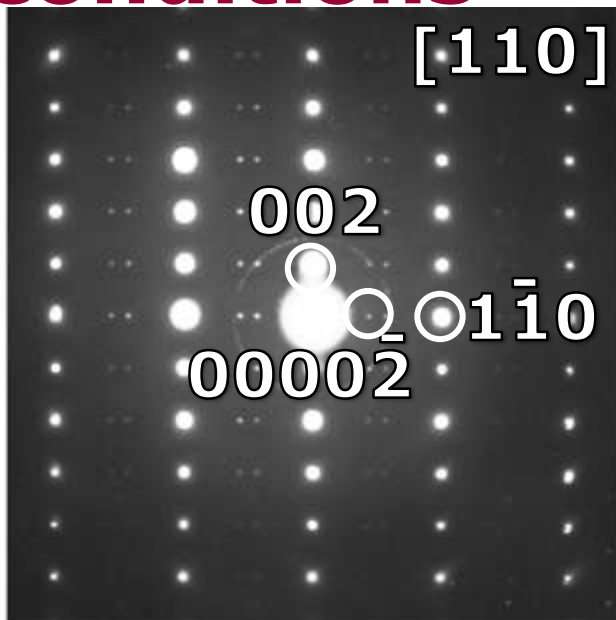
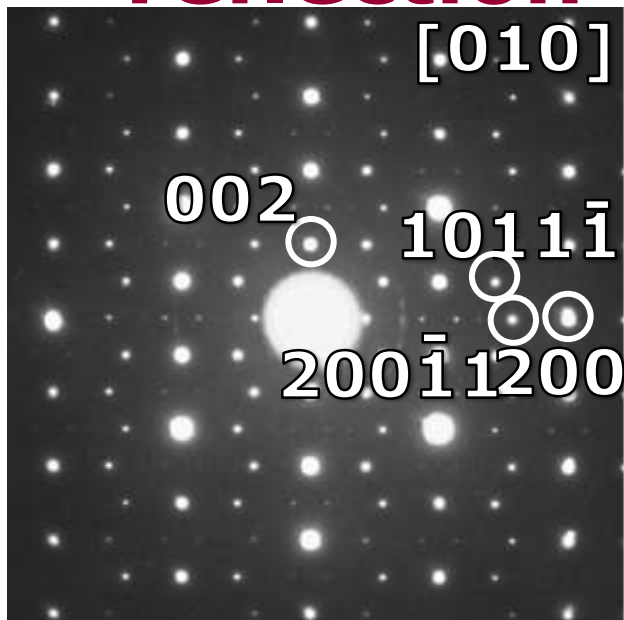


Could it be twinning and thus sufficient to use only one q-vector?

yes

no

Exercise 1-6: derive the reflection conditions



$$\left. \begin{array}{l} hkl: h+k+l=2n \\ hhl: l=2n \end{array} \right\}$$

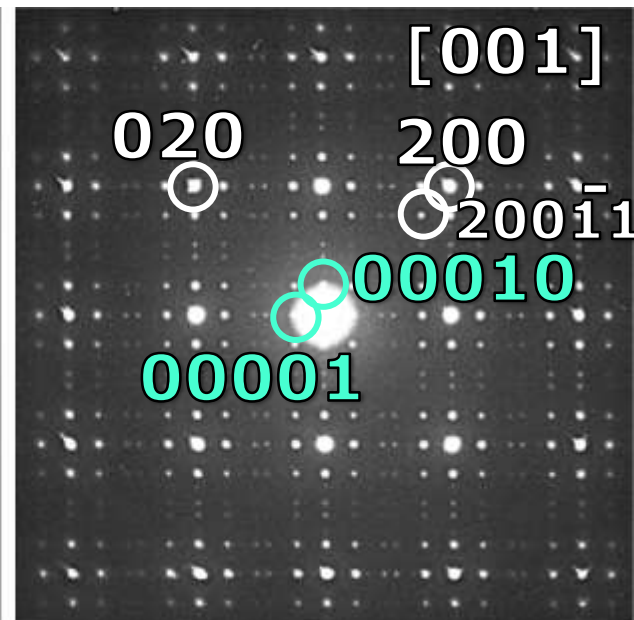
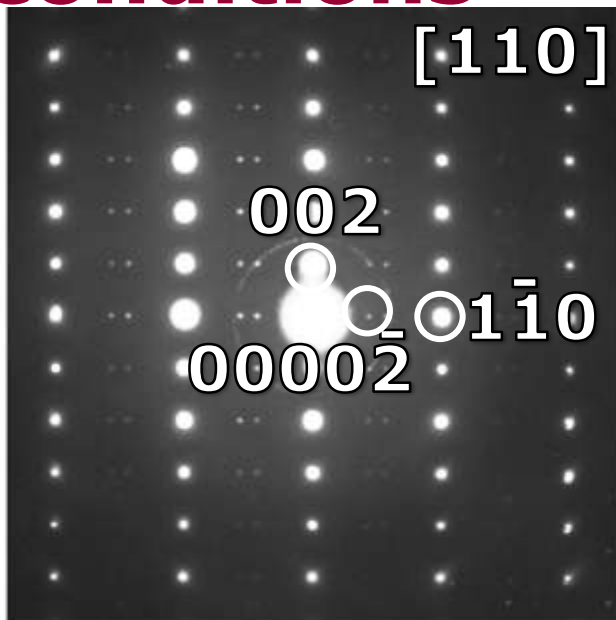
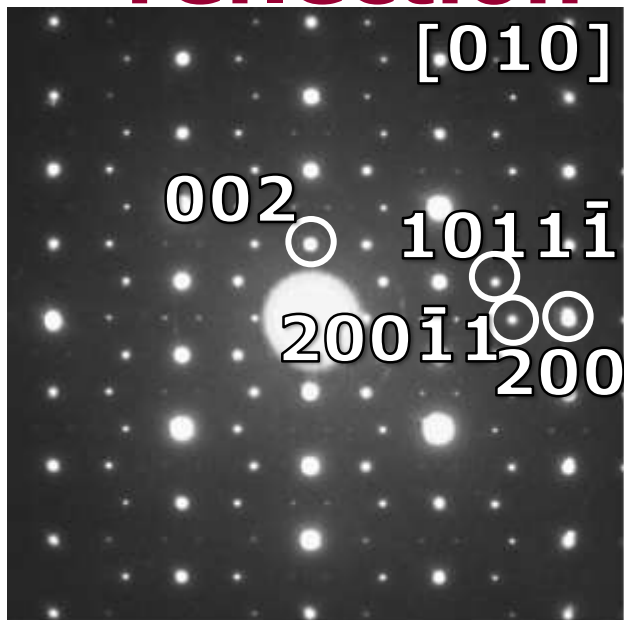
hklmn:

● $h+k+l+m+n=2i$

● $h+k+l=2i$

● $m+n=2i$

Exercise 1-6: derive the reflection conditions

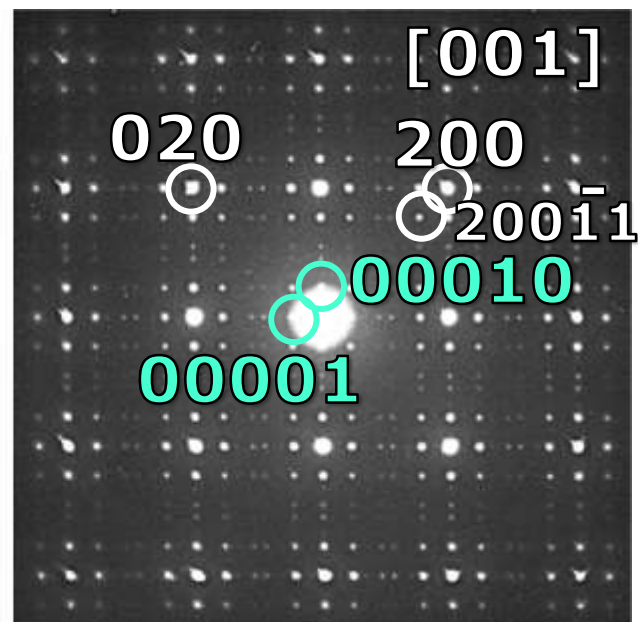
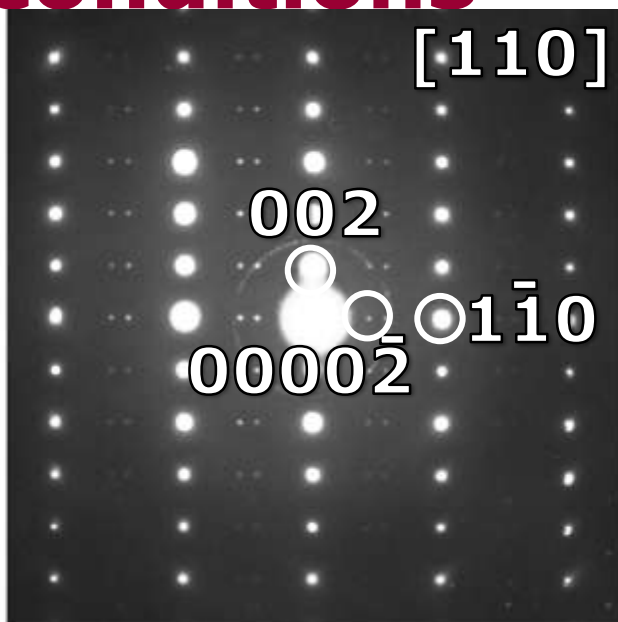
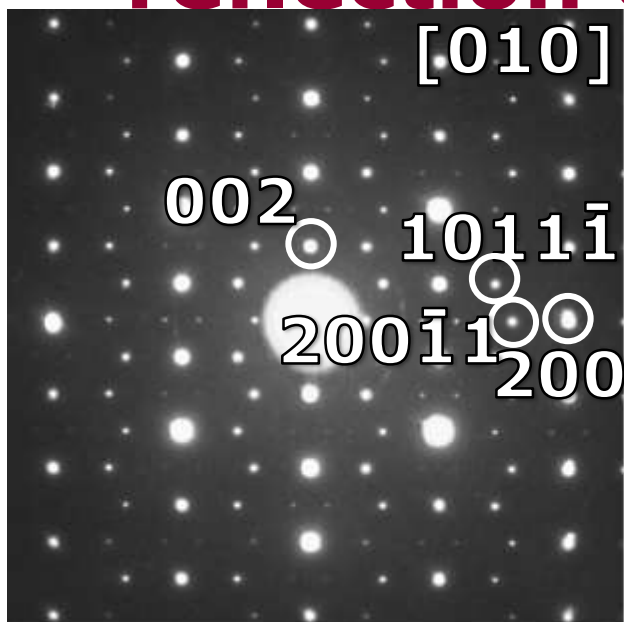


$$\left. \begin{array}{l} hkl: h+k+l=2n \\ hhl: l=2n \end{array} \right\}$$

hkln:

● $h+k+l=2i$

Exercise 1-6: derive the reflection conditions

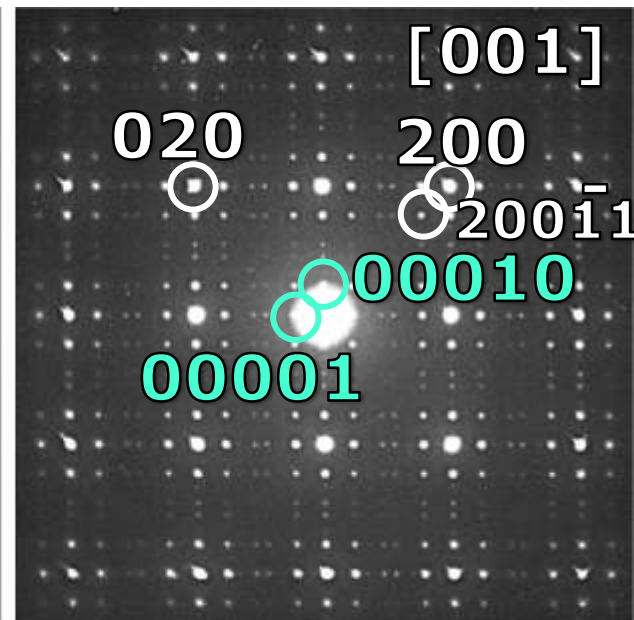
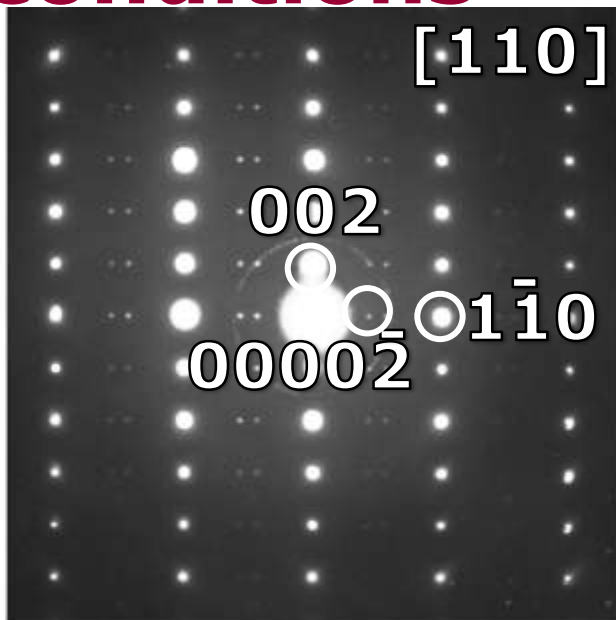
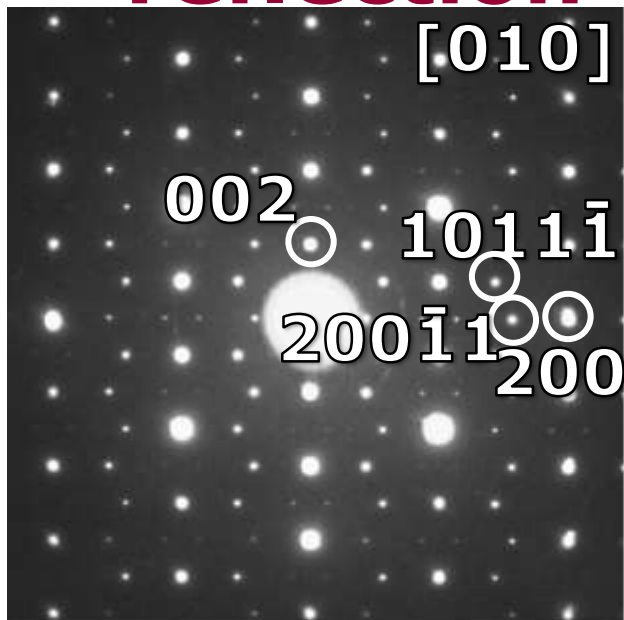


$$\left. \begin{array}{l}
 \text{hkl: } h+k+l=2n \\
 \text{hhl: } l=2n
 \end{array} \right\}$$

hhlm0:

- $l=2i$
- $h=2i$
- $l, m=2i$

Exercise 1-6: derive the reflection conditions

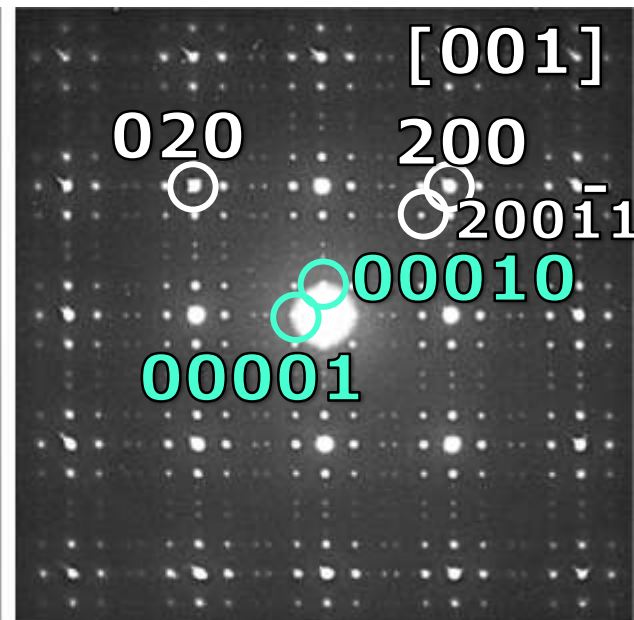
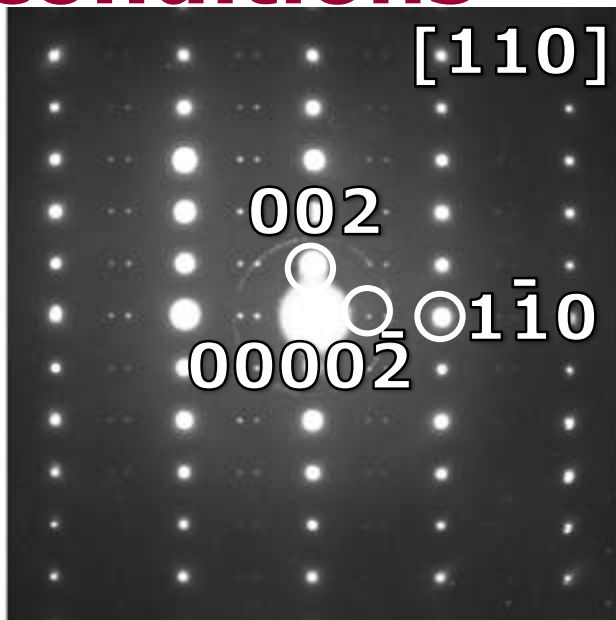
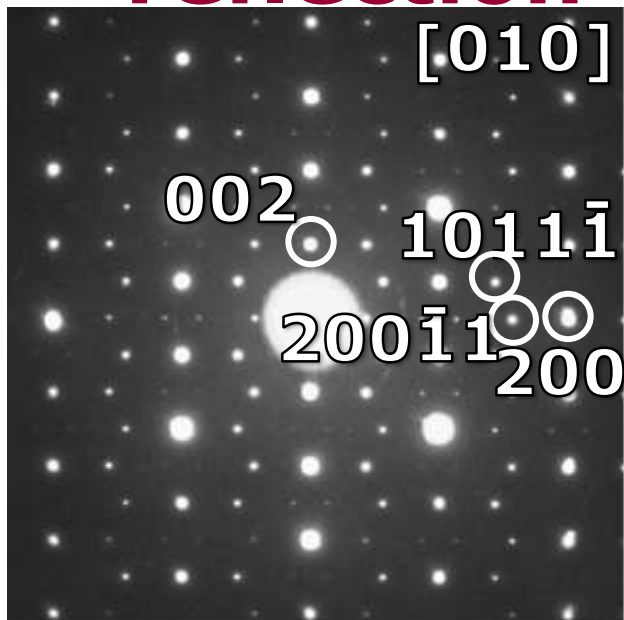


$$\left. \begin{array}{l} hkl: h+k+l=2n \\ hhl: l=2n \end{array} \right\} hlm0:$$

● $l, m=2i$

but
hhlm0: m=2i
is sufficient

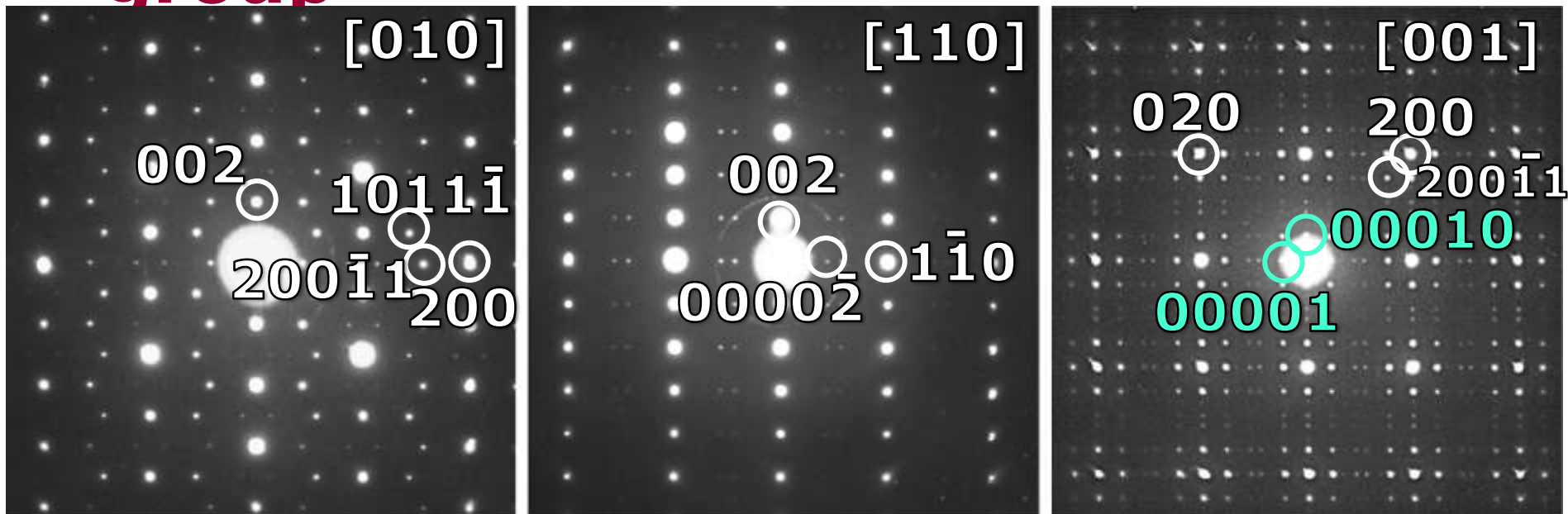
Exercise 1-6: derive the reflection conditions



$$\left. \begin{array}{l} hkl: h+k+l=2n \\ hhl: l=2n \end{array} \right\}$$

- $hklmn: h+k+l=2i$
- $hhlm0: m=2i$
($-hhl0n: n=2i$)

Determining the superspace group



- Online tables Yamamoto (Acta Cryst.A 1996, A52, 509)
- $I4/mmm(\alpha \alpha 0, -\alpha \alpha 0)00mg$

Use of doing ED

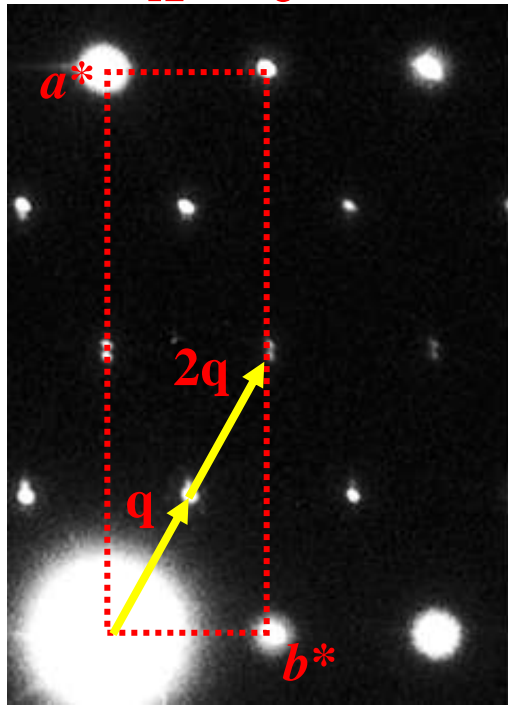
- Deriving approximate cell parameters, modulation vector and superspace group
- Not useful for: precise parameters, precise modulation vector
- Excellent if multiphase sample or satellites very weak in XRD/ND
- Be aware of:
 - Twinning
 - Multiple diffraction (for forbidden refls., for using intensities,...)
 - The possibility of missing important sections of reciprocal space



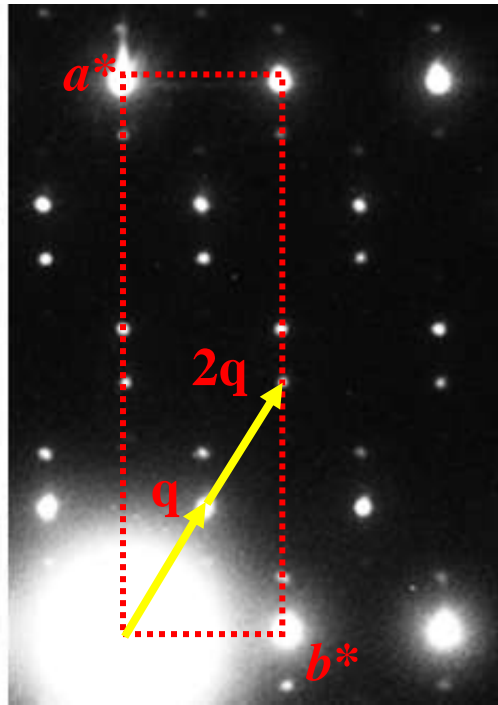
Relation composition – modulation vector

- Most TEM have EDX available for composition determination
- Determination of direct links between composition and modulation vector, also in multiphased samples

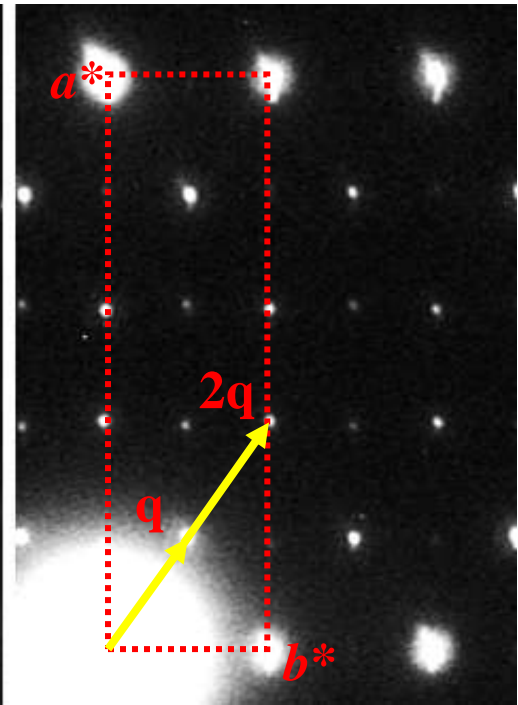
$x = 0$



$x = 0.2$

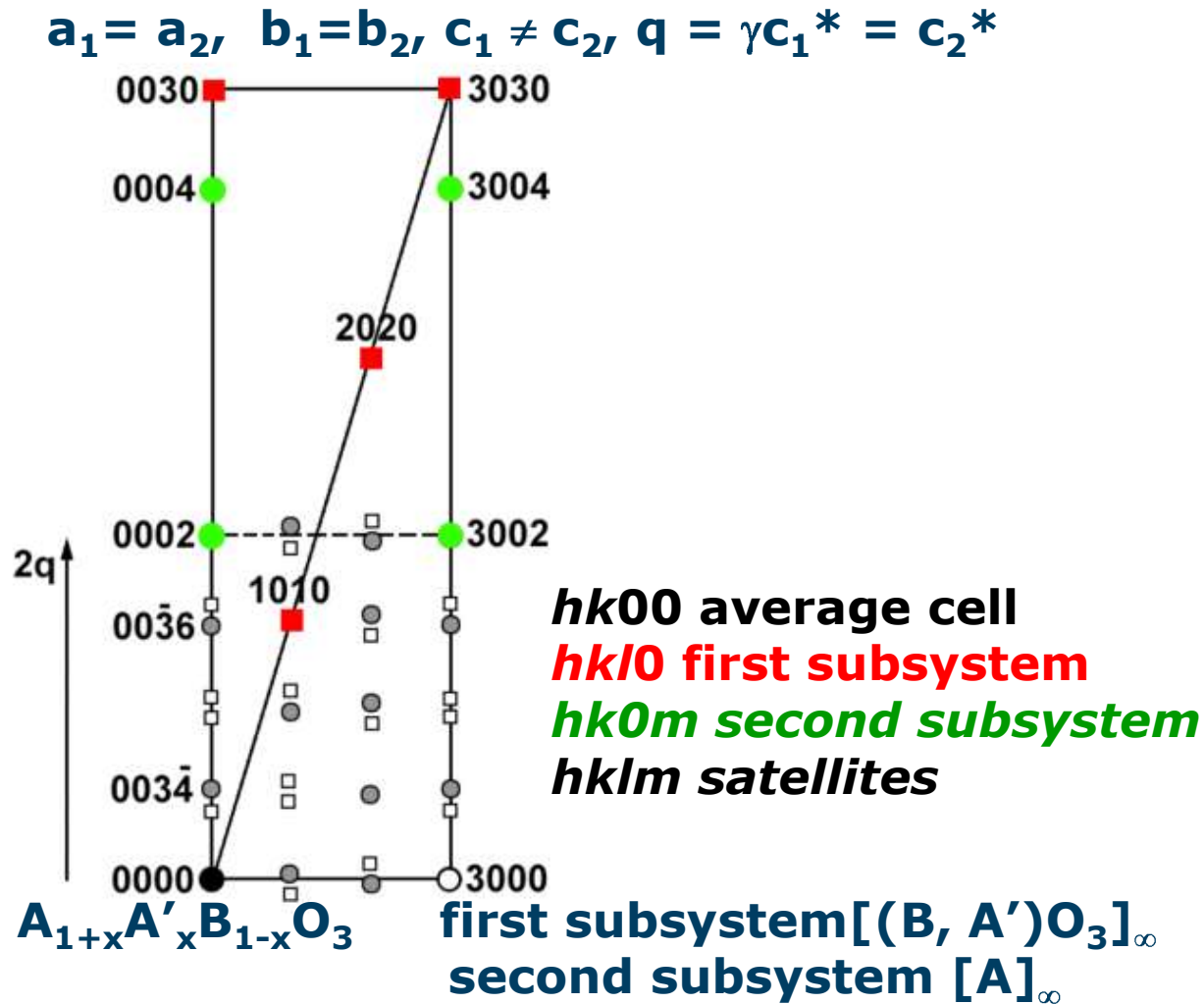
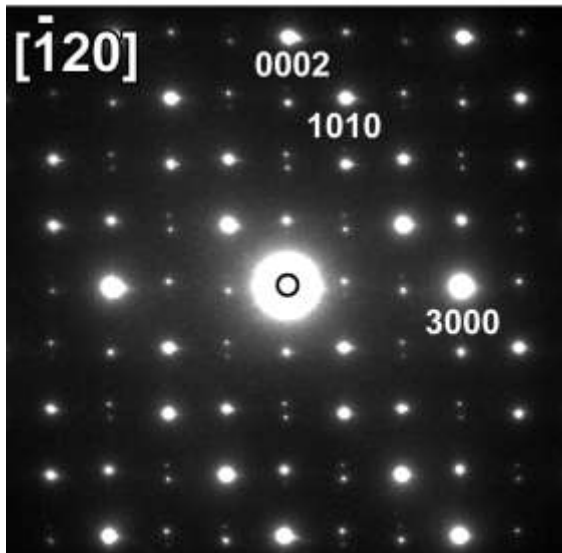
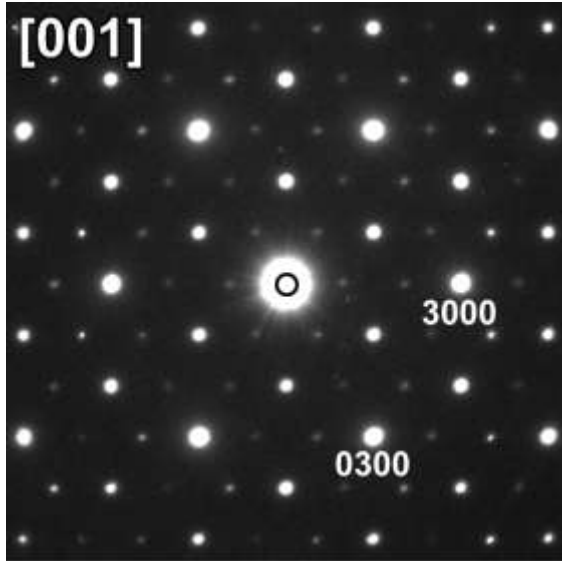


$x = 0.4$



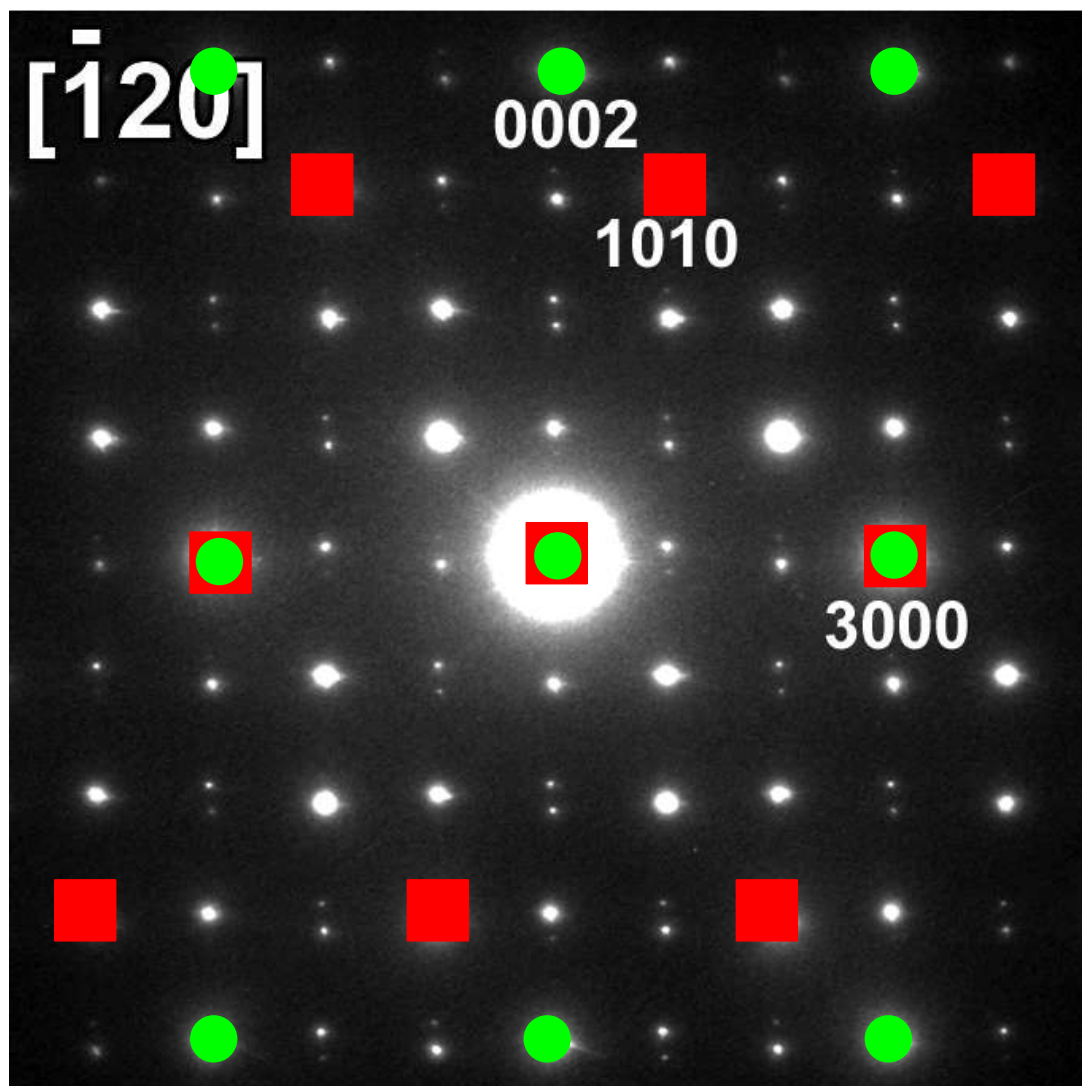
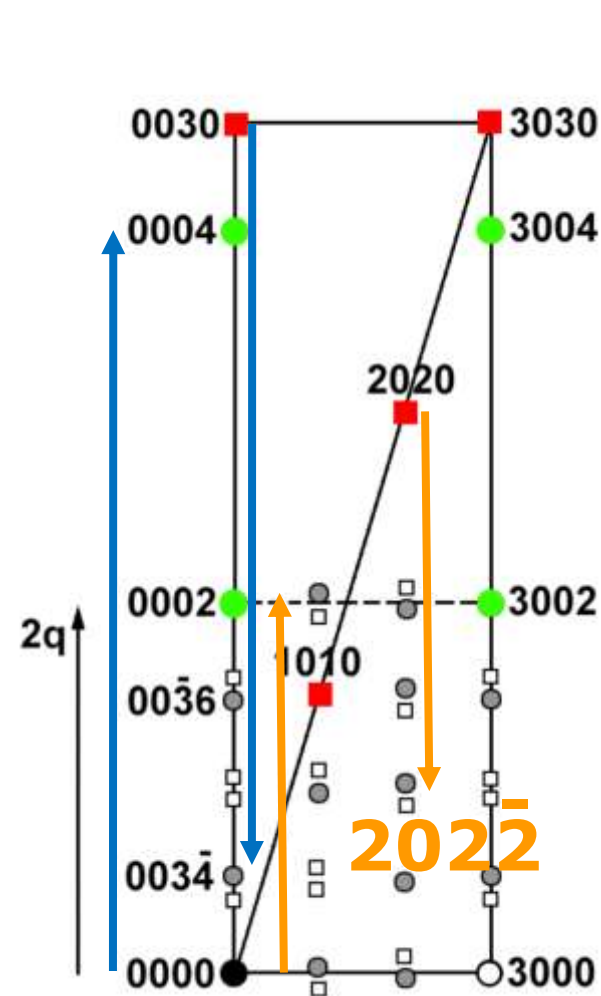
(Abakumov et al., Chem. Mater. 20 (13), pp. 4457-4467)

ED of a composite structure (CS)

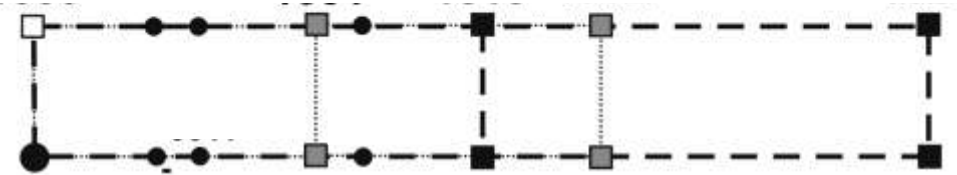
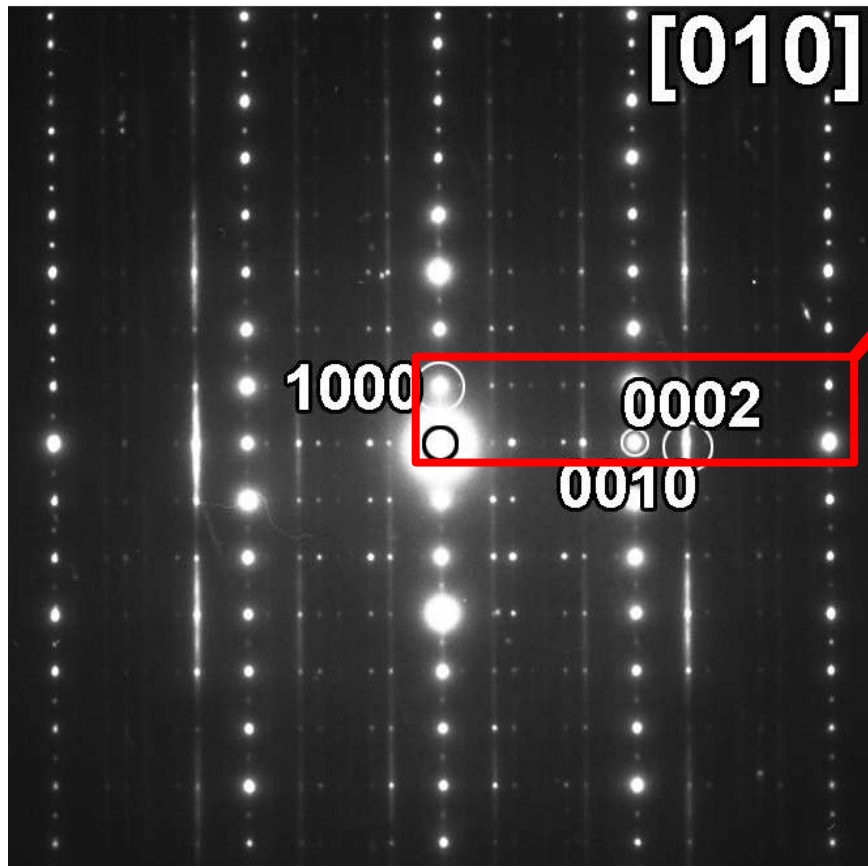


Mandal et al., Chem. Mater., 19, 25 (2007) 6158

ED of a composite structure (CS)



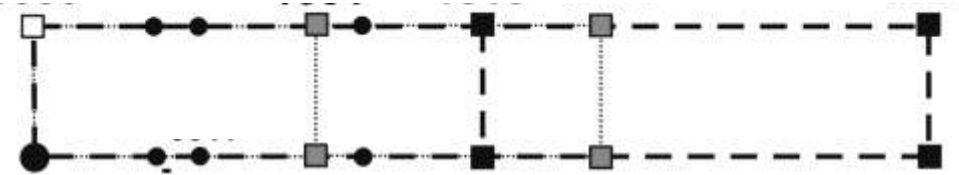
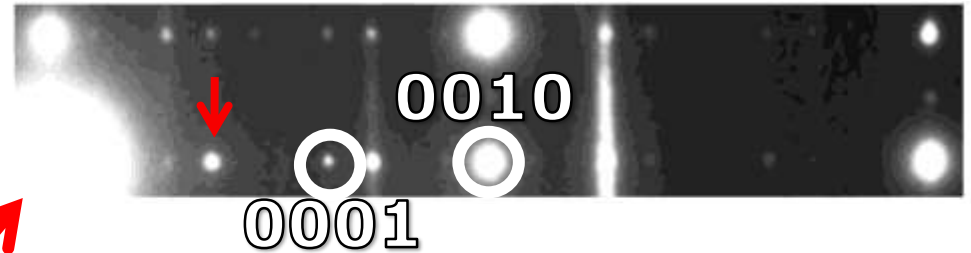
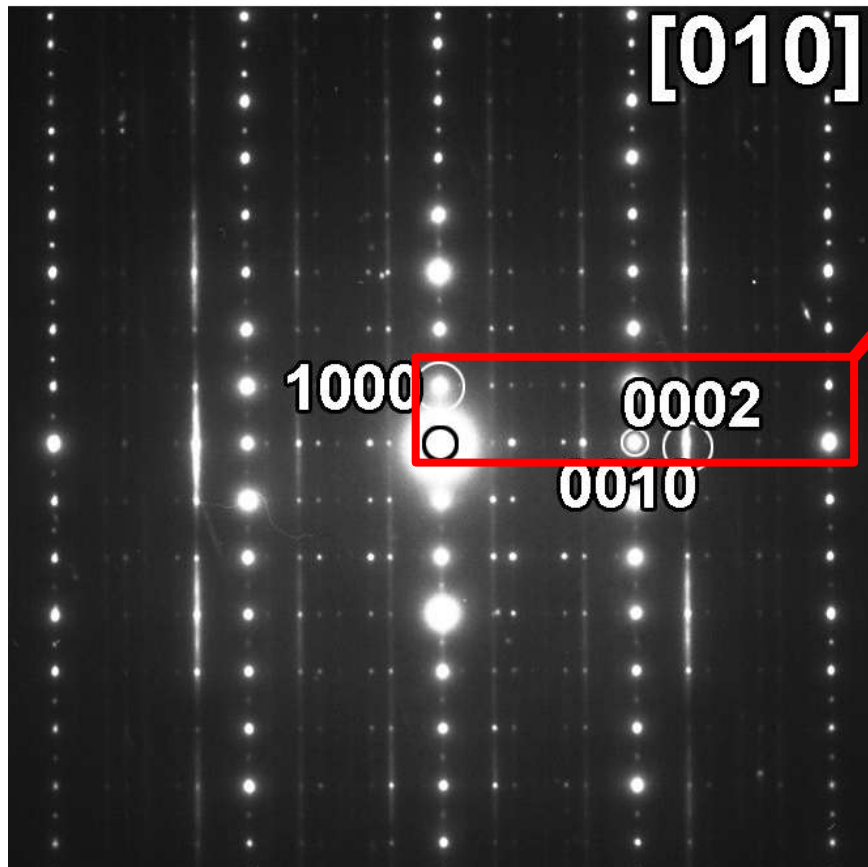
Exercise 2: composite structure: index the pattern



Which reflection is $001\bar{1}$?



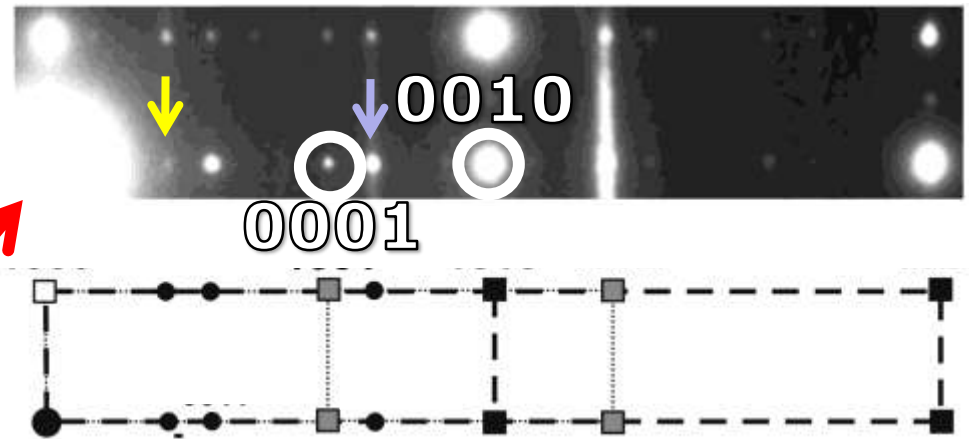
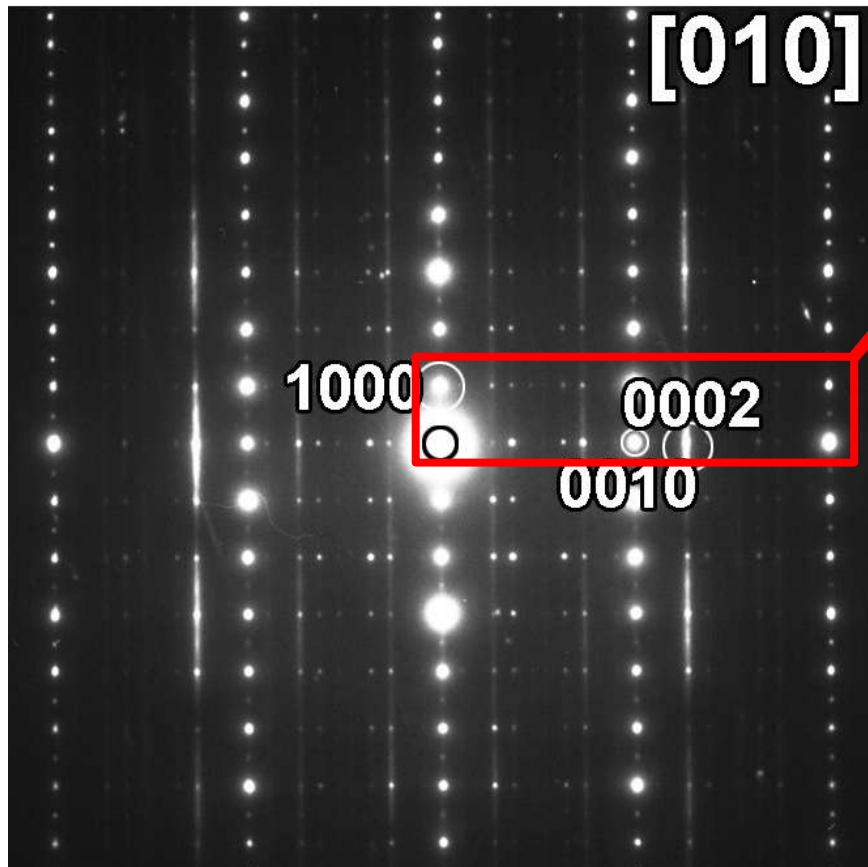
Exercise 2: composite structure: index the pattern



Which reflection is $001\bar{1}$?



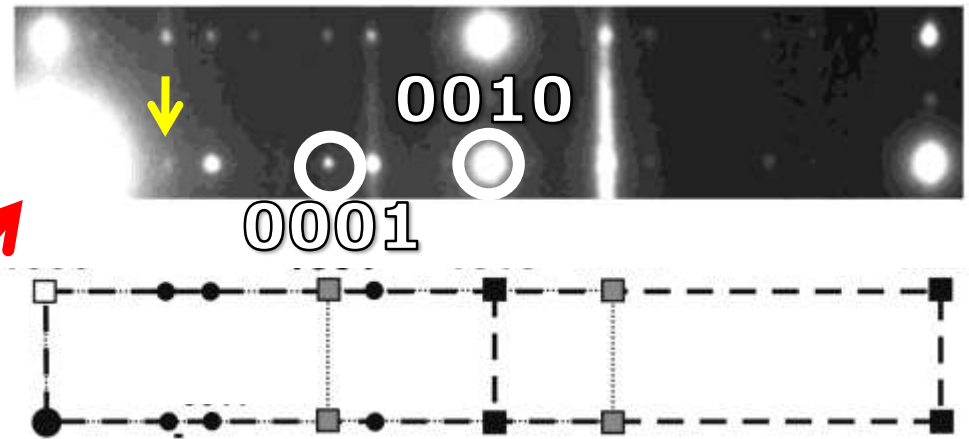
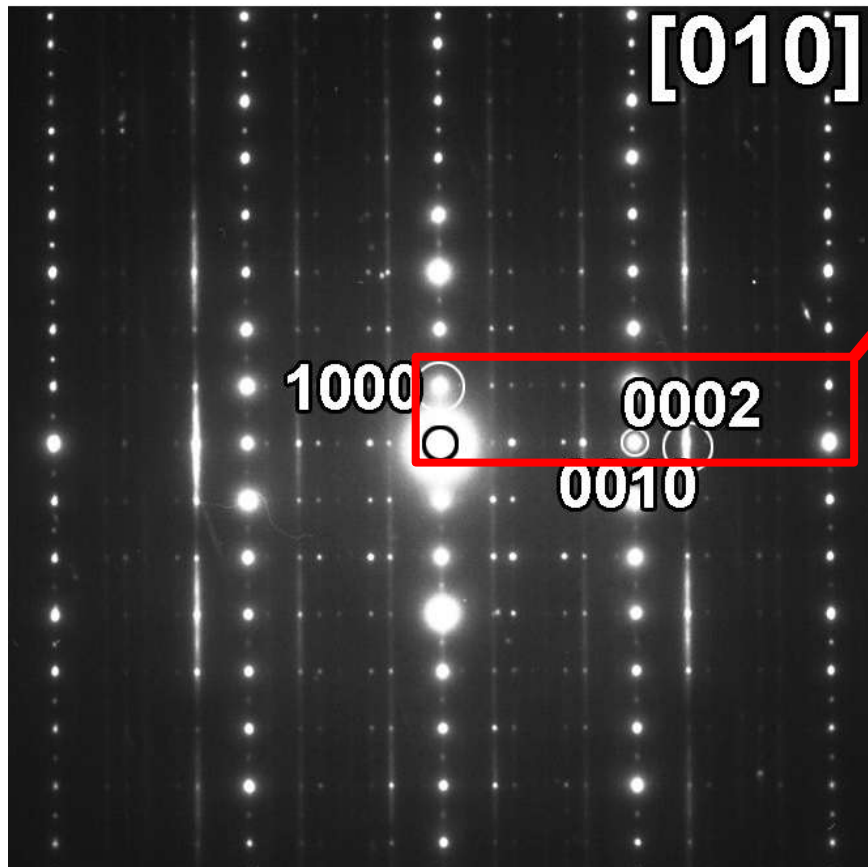
Exercise 2: composite structure: index the pattern



Which reflection is $00\bar{1}2$?



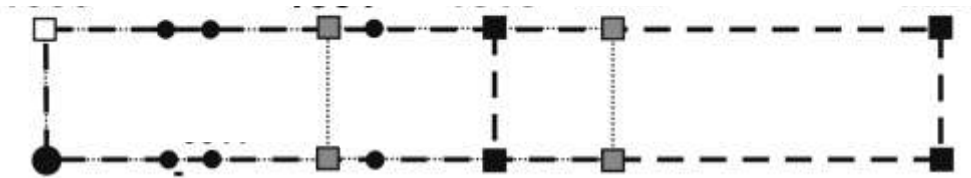
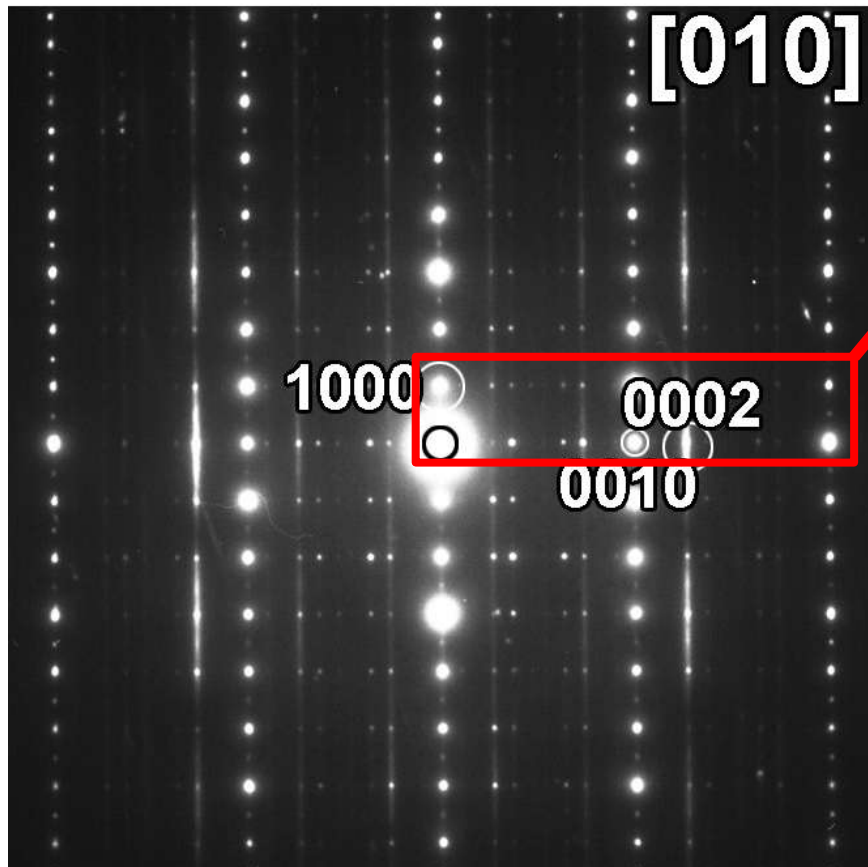
Exercise 2: composite structure: index the pattern



Which reflection is $00\bar{1}2$?



Exercise 2: composite structure: index the pattern

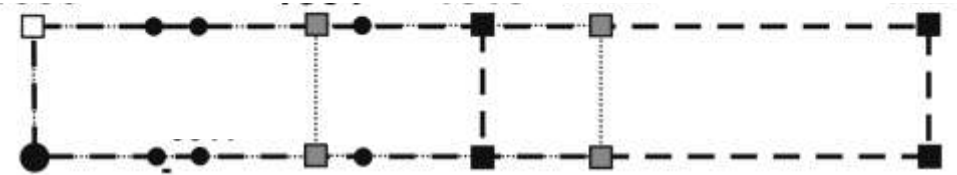
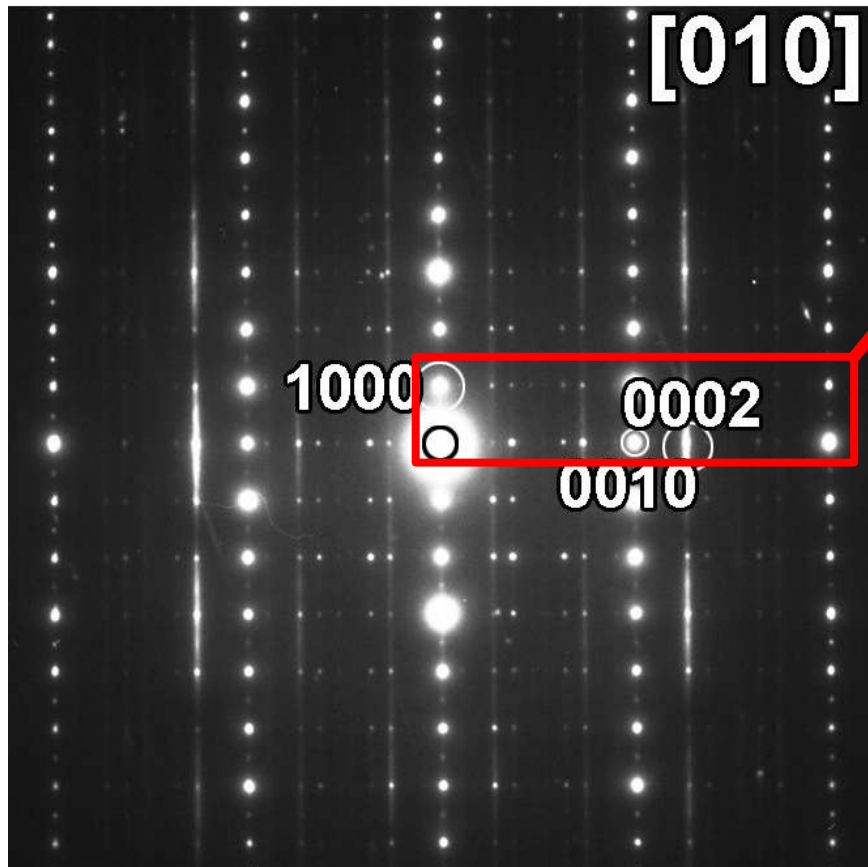


Which reflection does the blue arrow point at?

- 002 $\bar{2}$ ●
- 00 $\bar{2}$ 1 ●
- 001 $\bar{2}$ ●



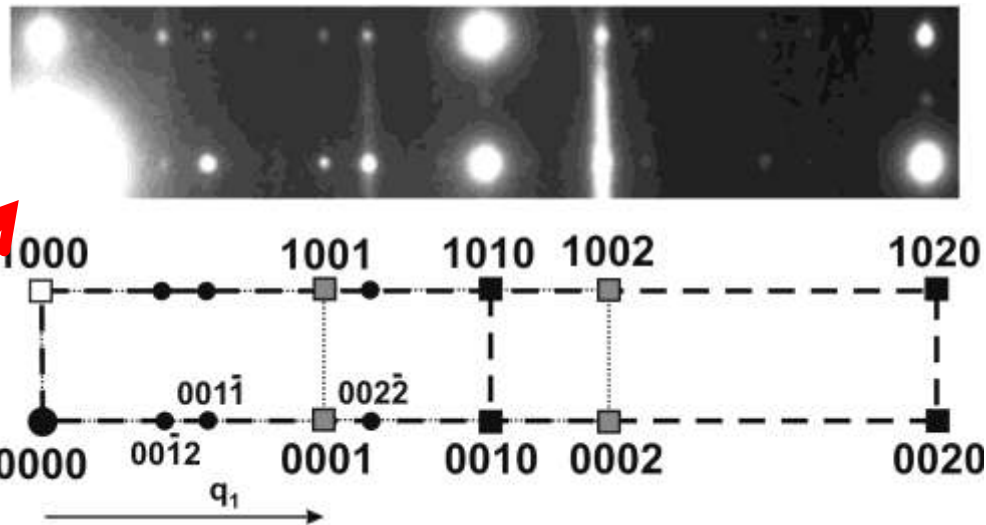
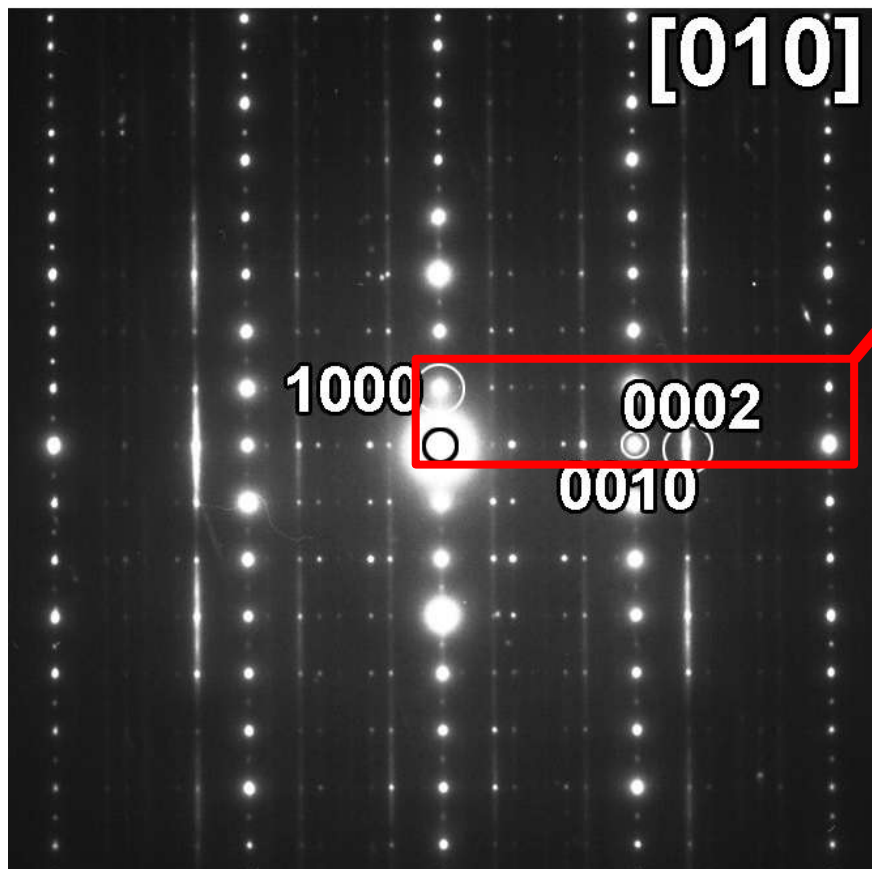
Exercise 2: composite structure: index the pattern



Which reflection does the blue arrow point at?
 $002\bar{2}$ ●

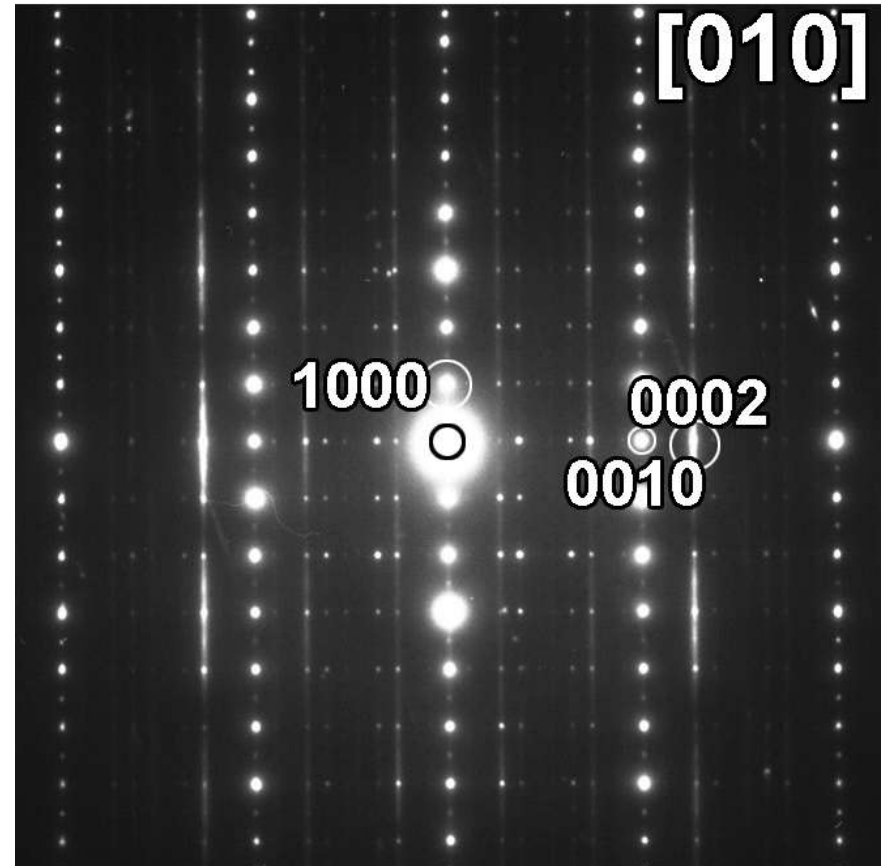


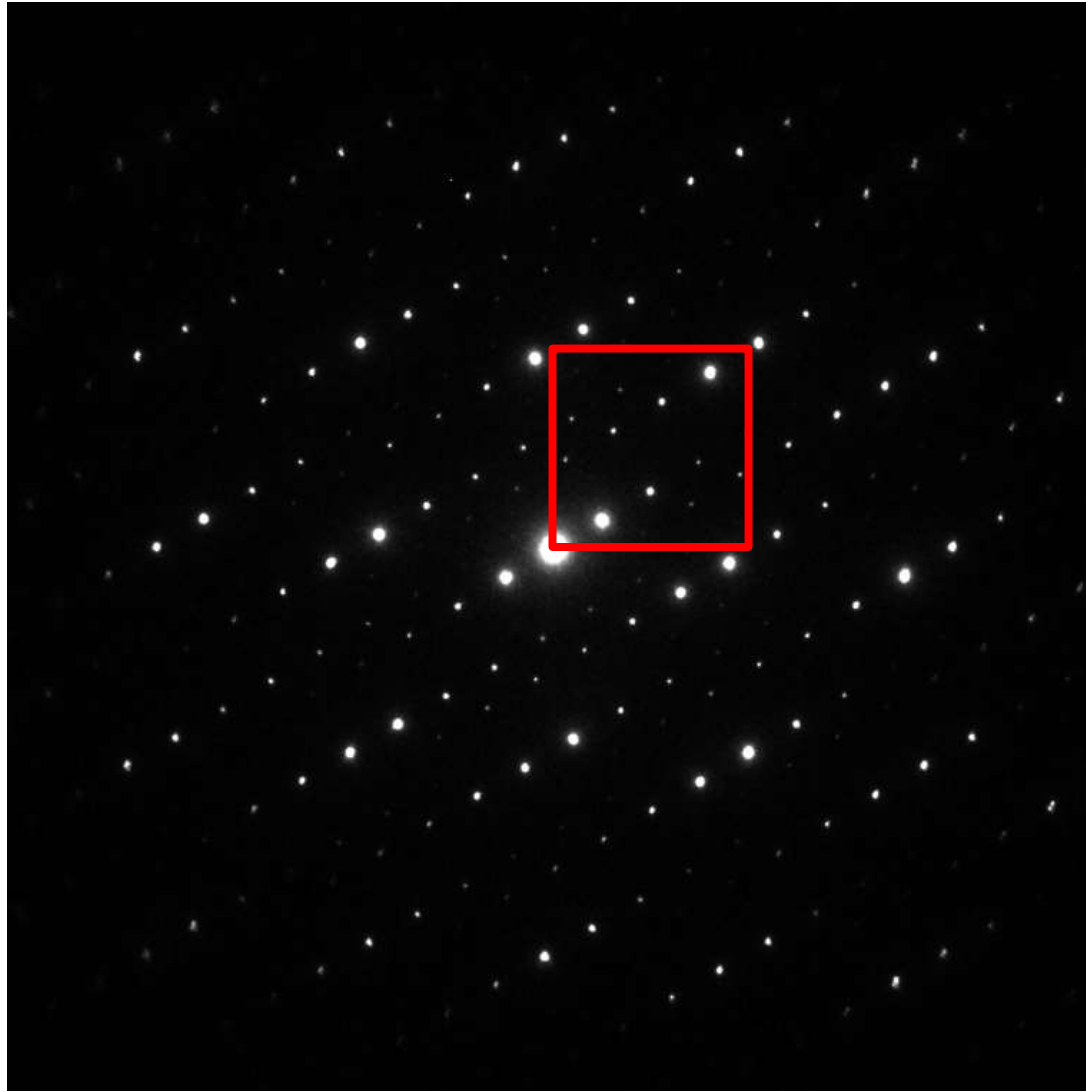
Exercise 2: composite structure: index the pattern - solution

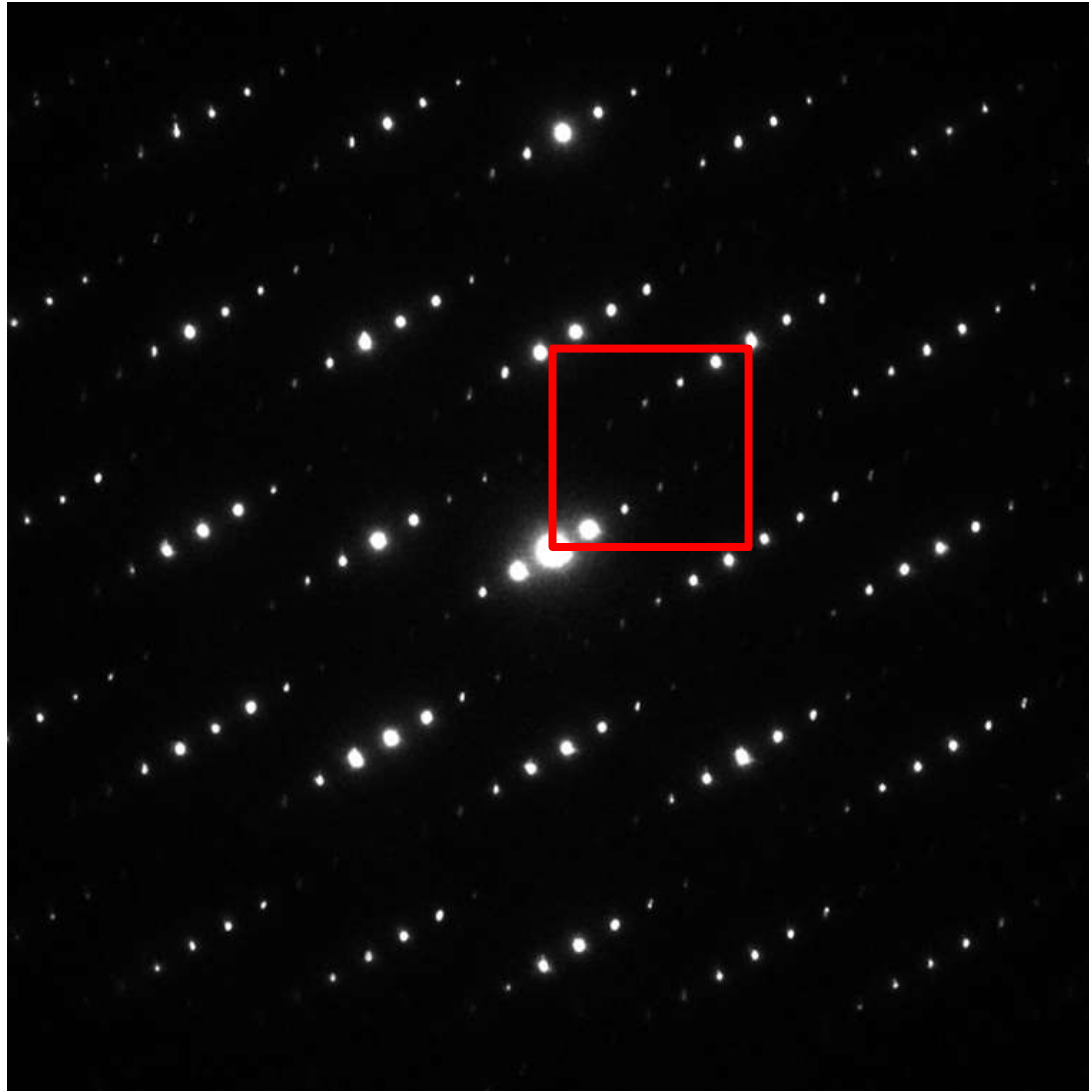


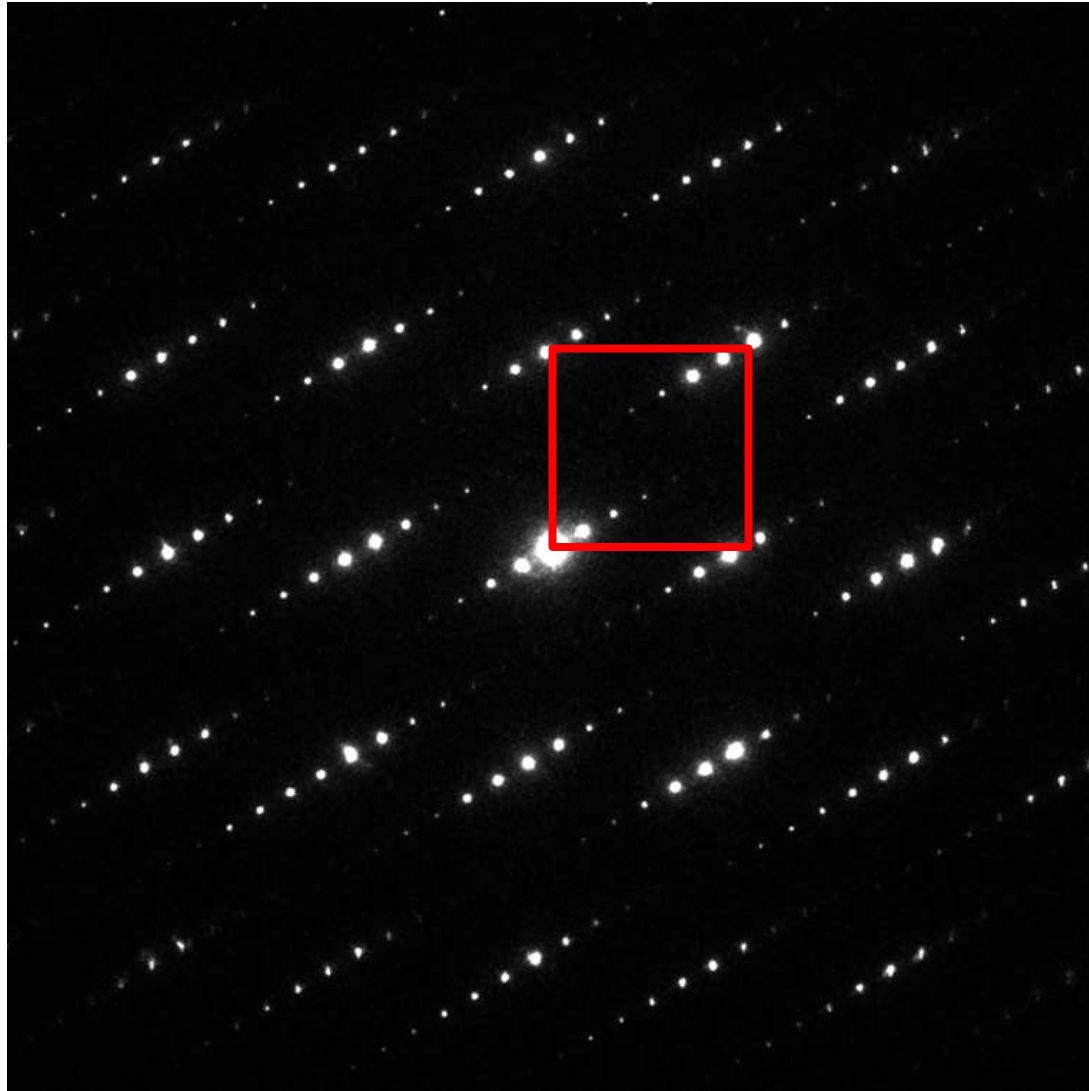
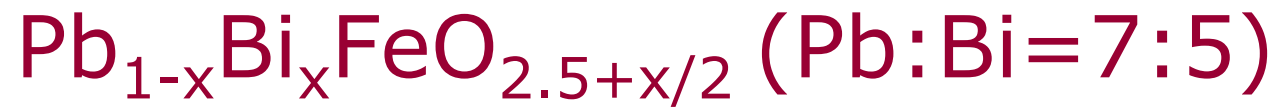
Streaks

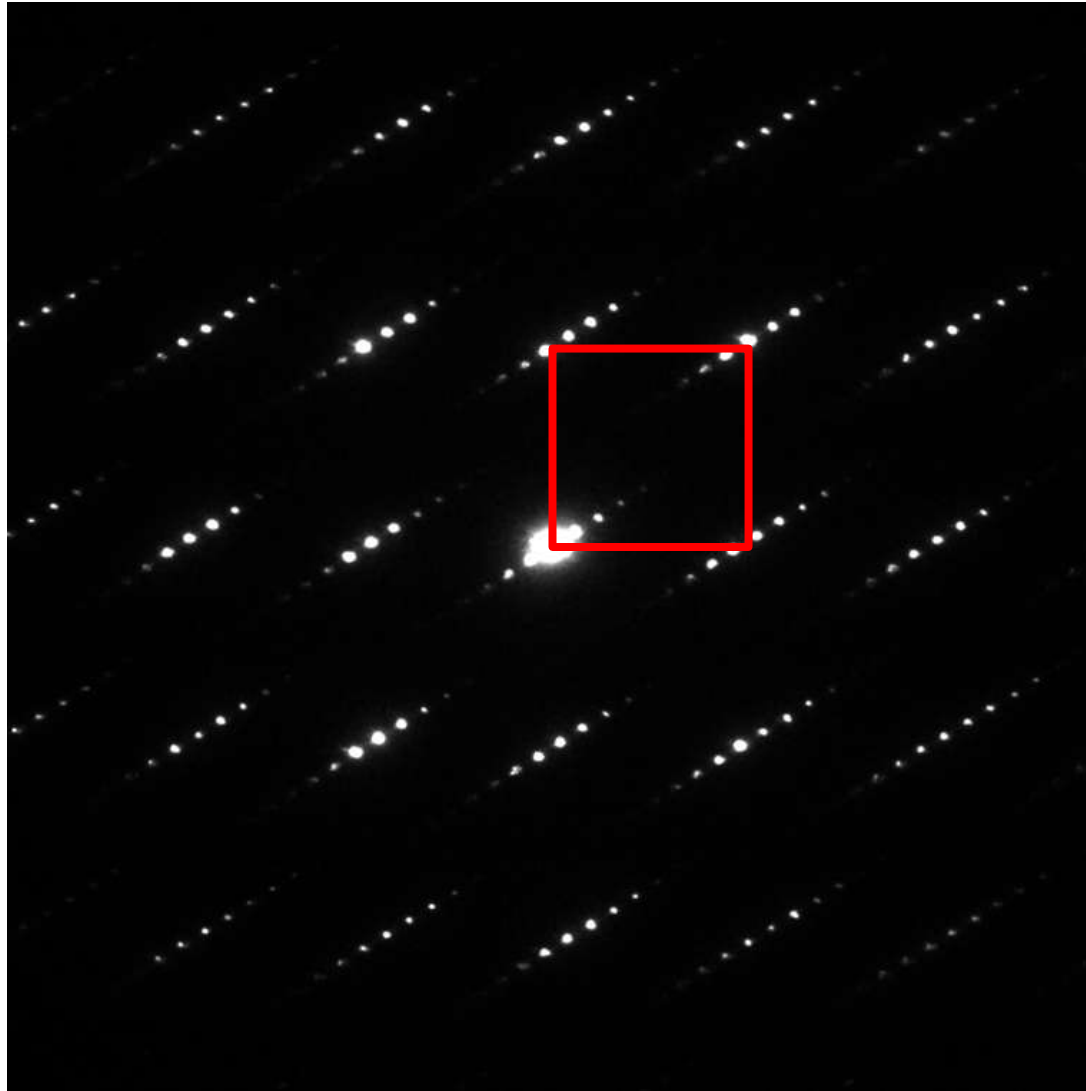
- order in the plane,
disorder between
these planes: streak
on ED pattern
- disorder in two
directions: diffuse
intensity plane: streak
or background
intensity depending on
the intersection



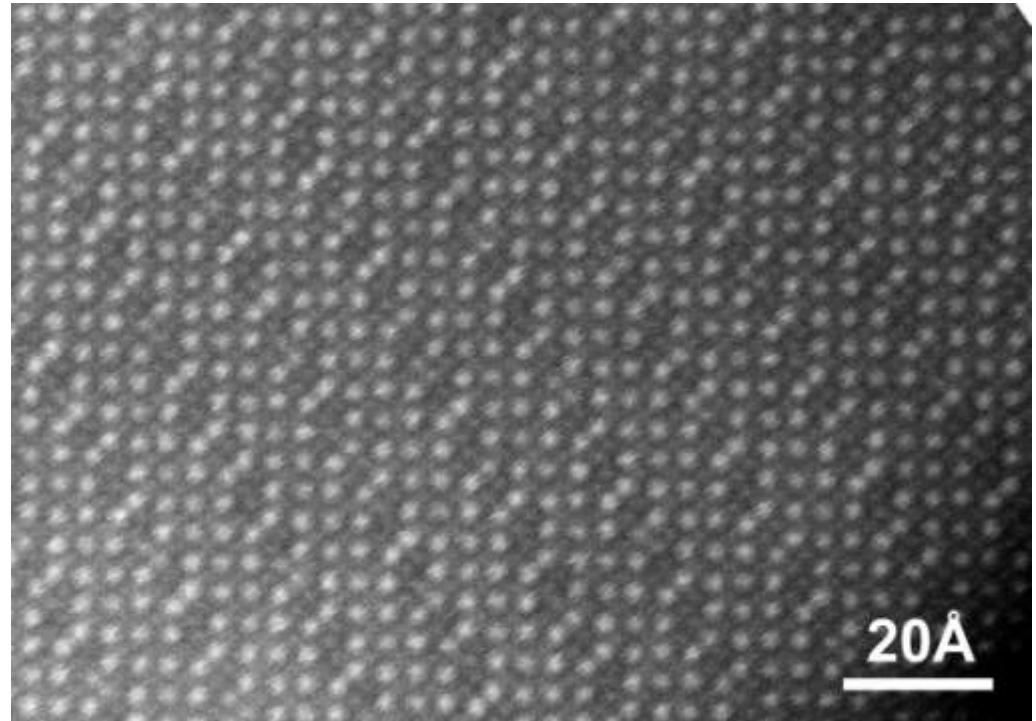
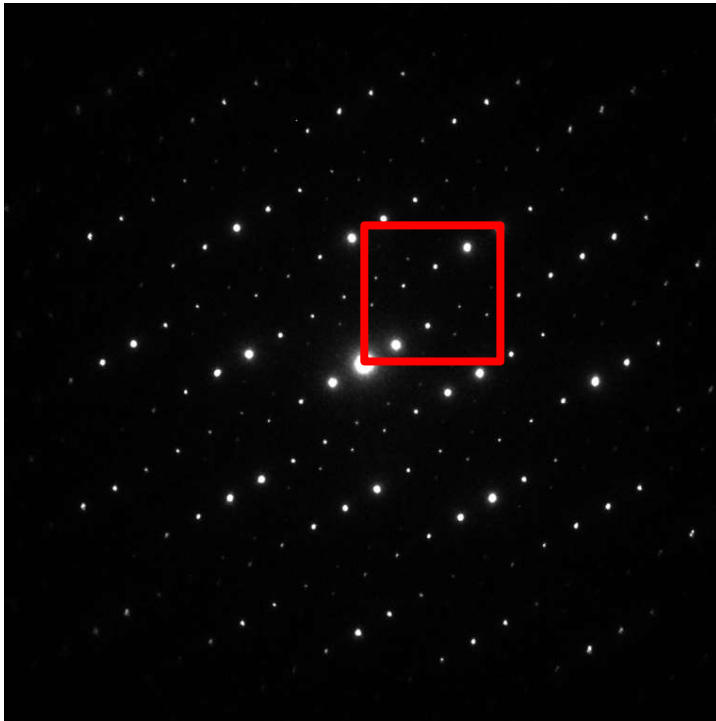




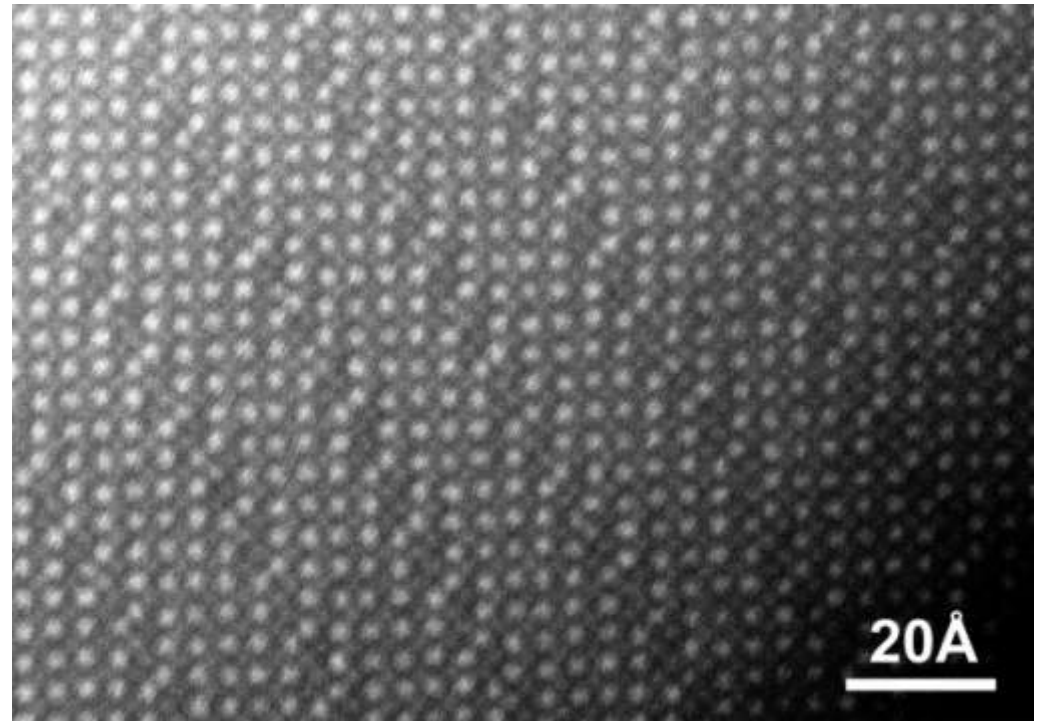
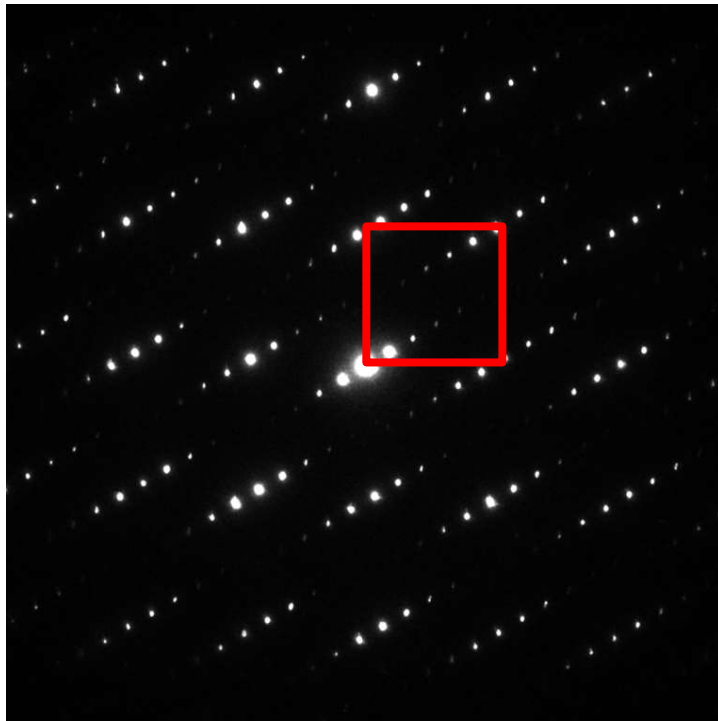


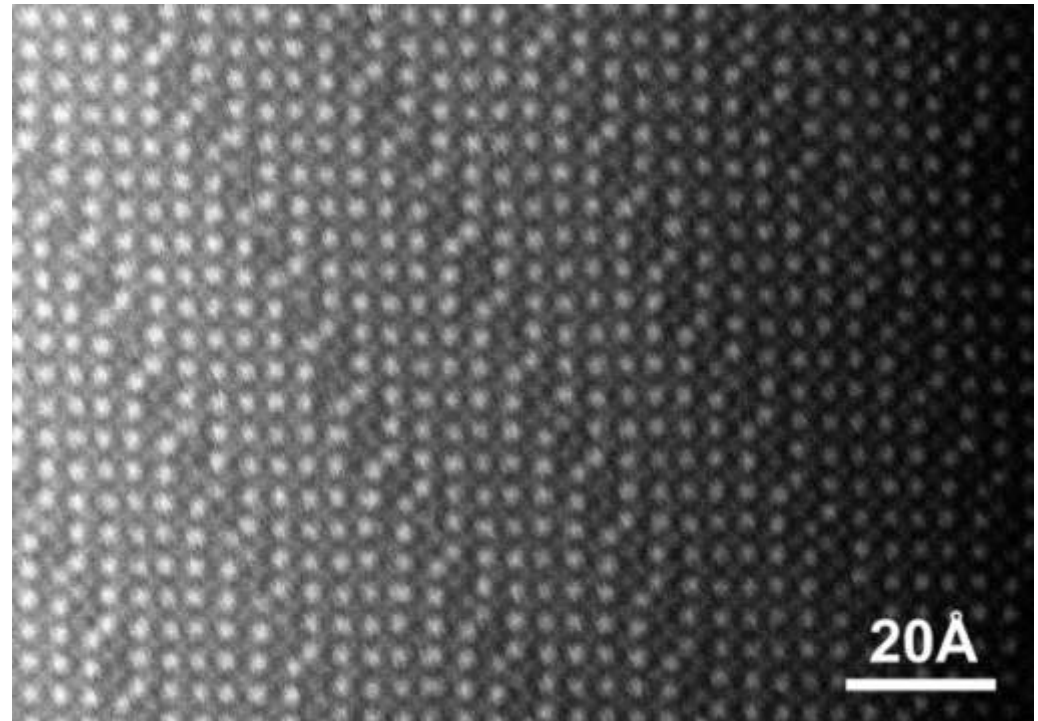
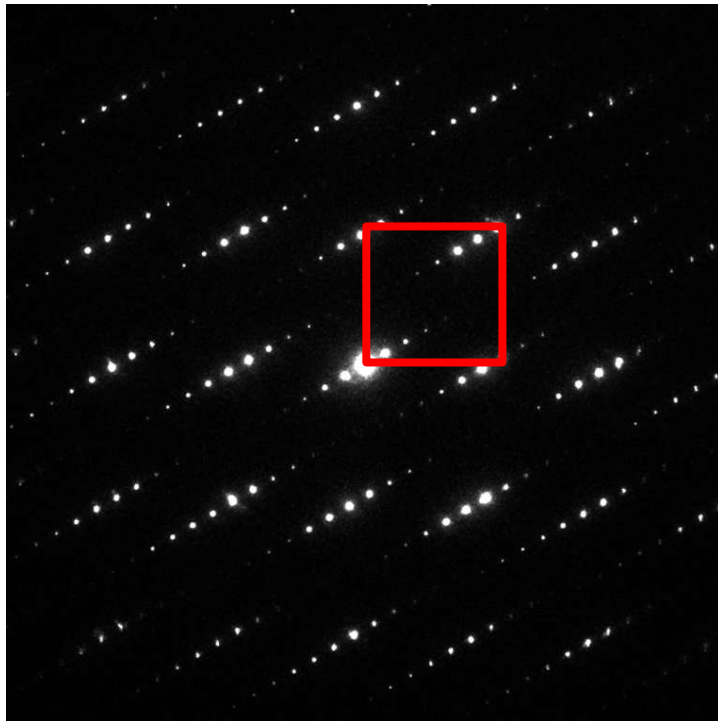
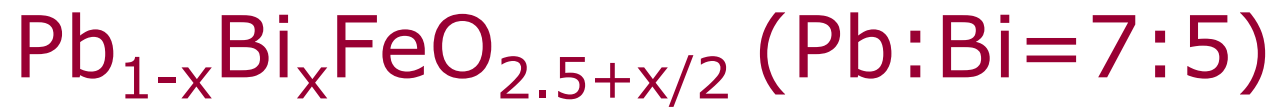


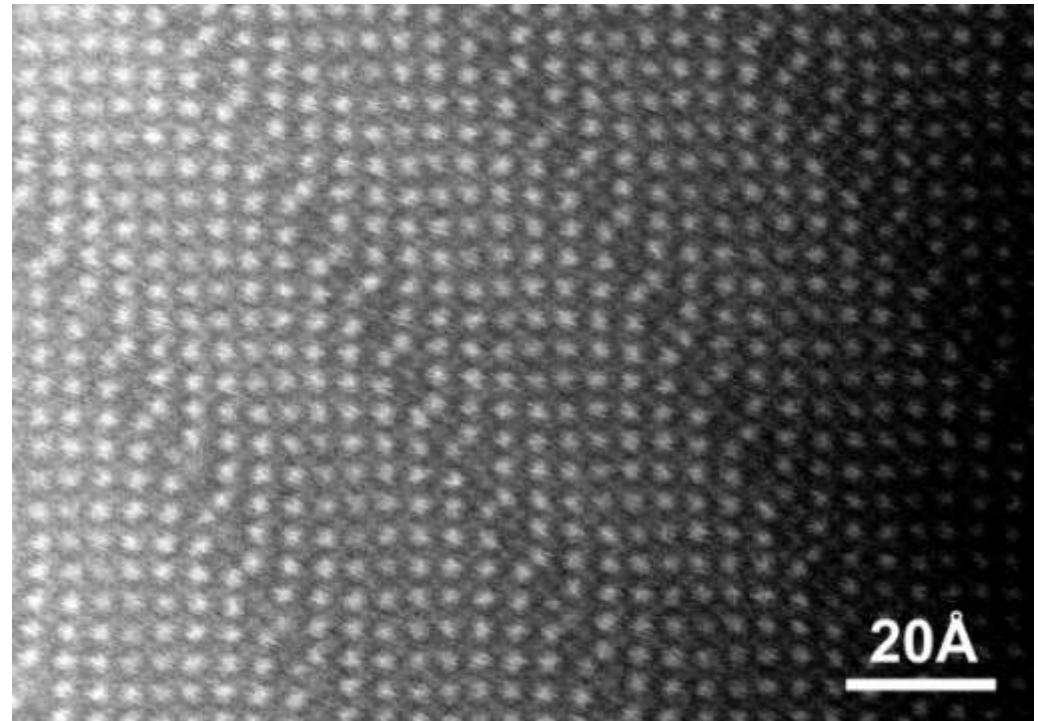
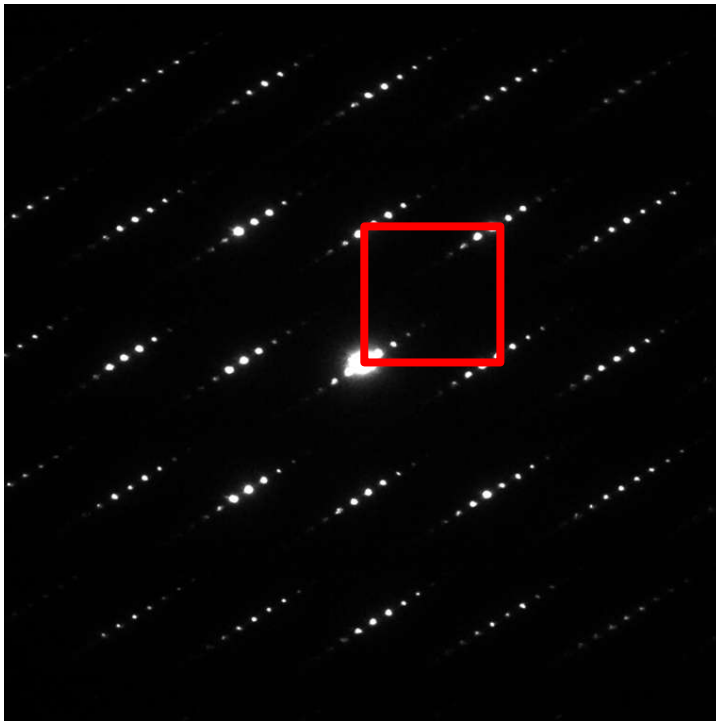
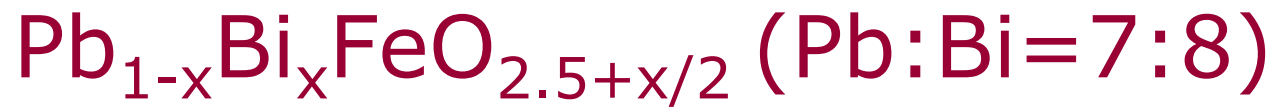
$\text{Pb}_{1-x}\text{Bi}_x\text{FeO}_{2.5+x/2}$ (Pb:Bi=7:0.5)



$\text{Pb}_{1-x}\text{Bi}_x\text{FeO}_{2.5+x/2}$ (Pb:Bi=7:3)







HRTEM and HAADFSTEM-images

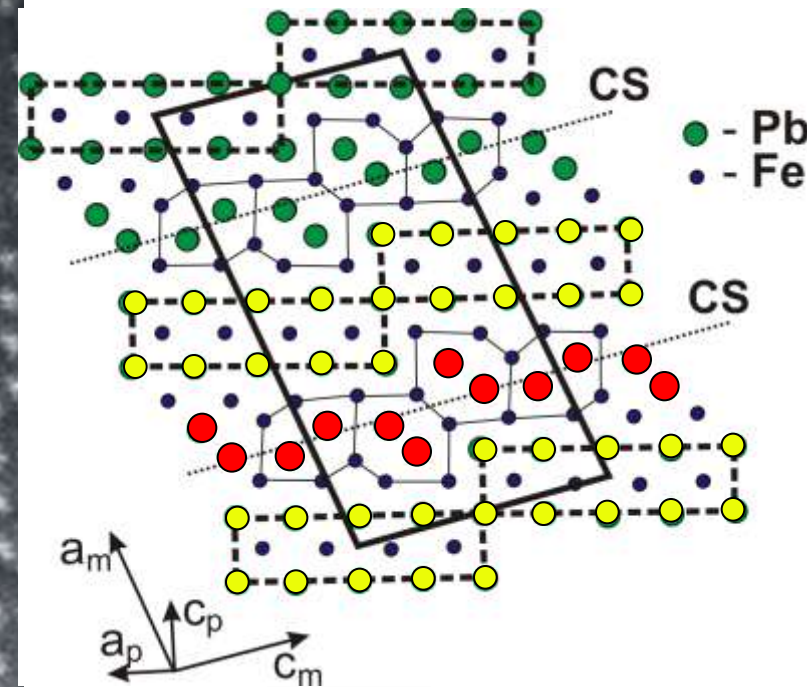
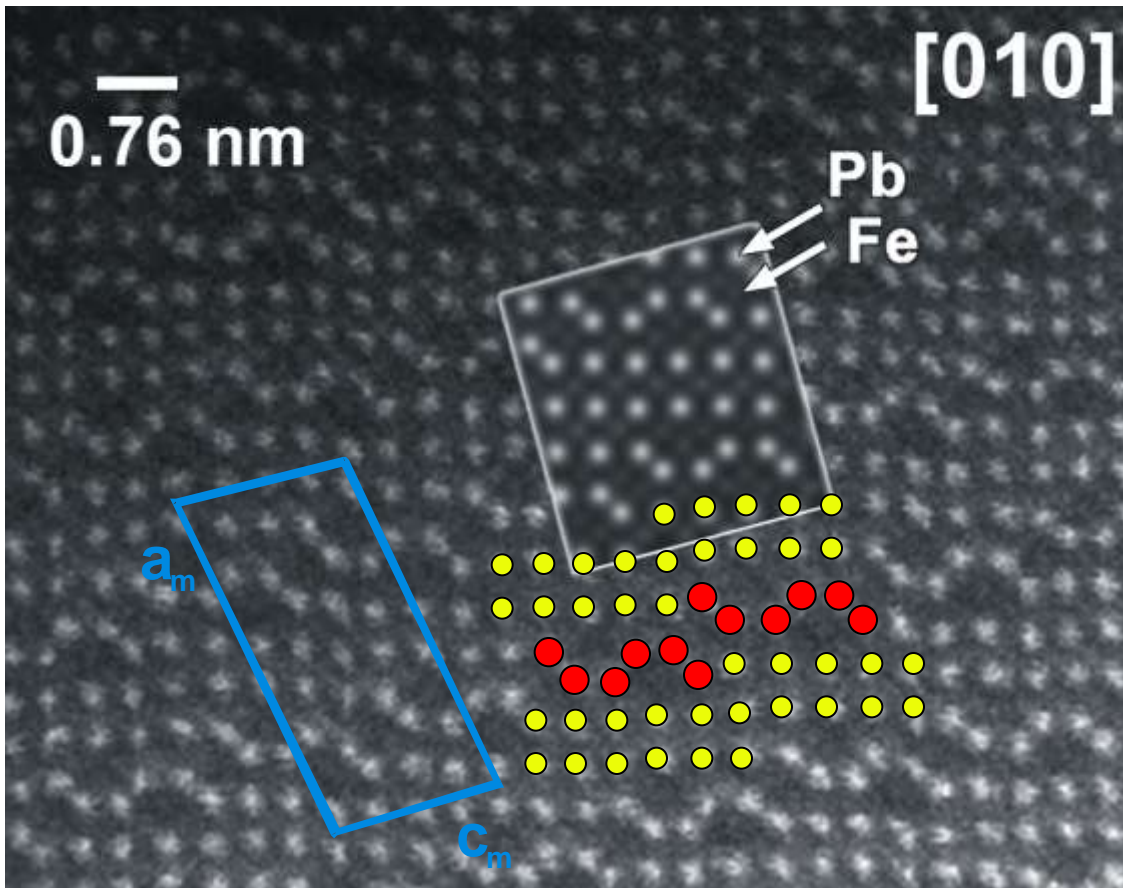
HAADF-STEM

- contrast related to Z^n
- direct information about the heavy atom positions
- brighter with thickness also
- correct orientation very important!
- scanned image: positions imprecise

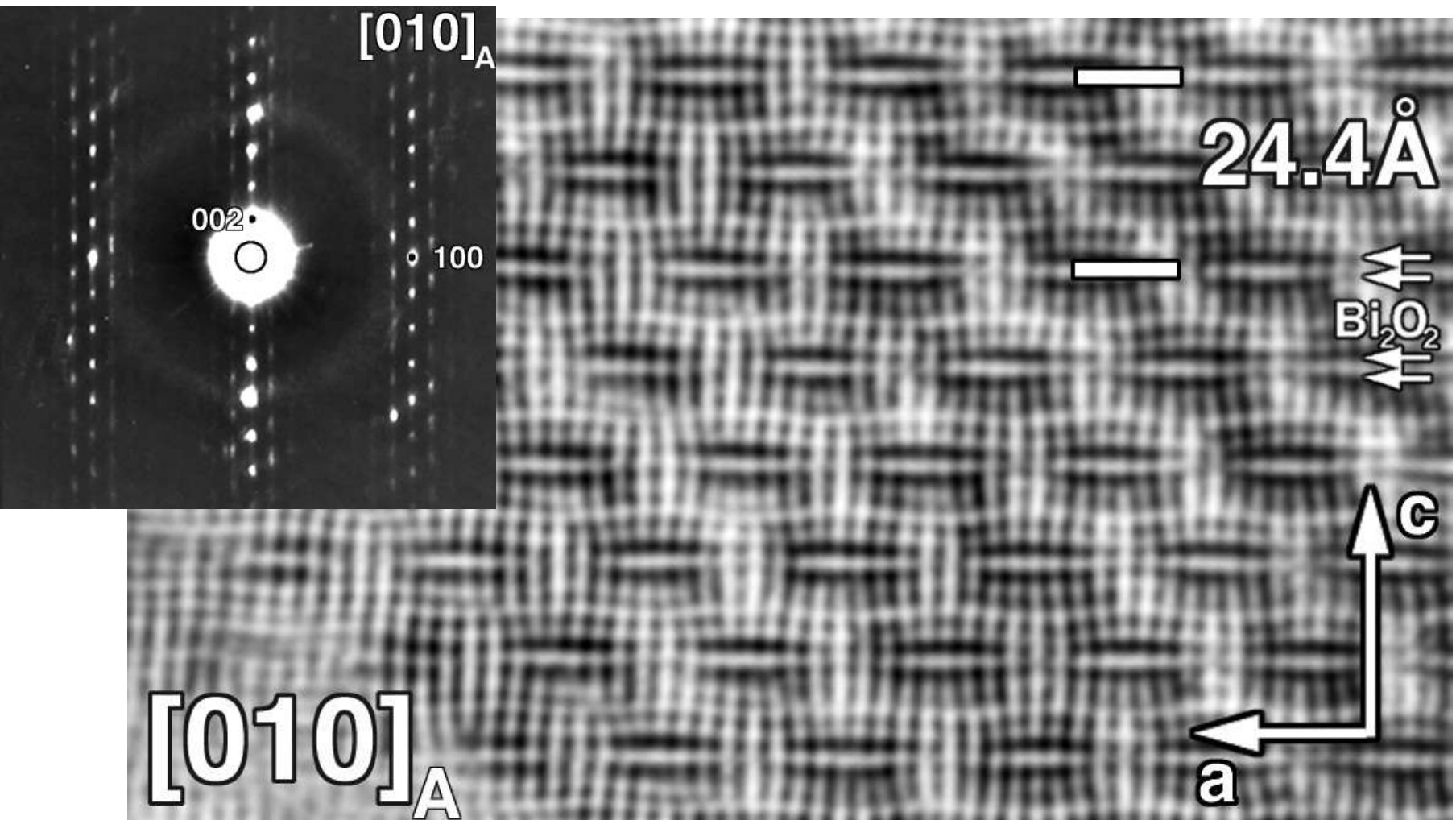
HRTEM

- complicated phase transfer function
- no direct interpretation possible
- contrast changes with thickness and defocus
- correct orientation very important!
- one-shot image: position of the contrast objects reliable

Heavy atoms from HAADF-STEM



High resolution transmission electron microscopy (HREM)



 $\text{Bi}_2\text{Sr}_{1.2}\text{La}_{0.8}\text{CuO}_{6.46-x}\text{F}_{2x}$ Hadermann et al., JSSC, 156, 2 (2001) 455-451

Relation structure – image in HRTEM

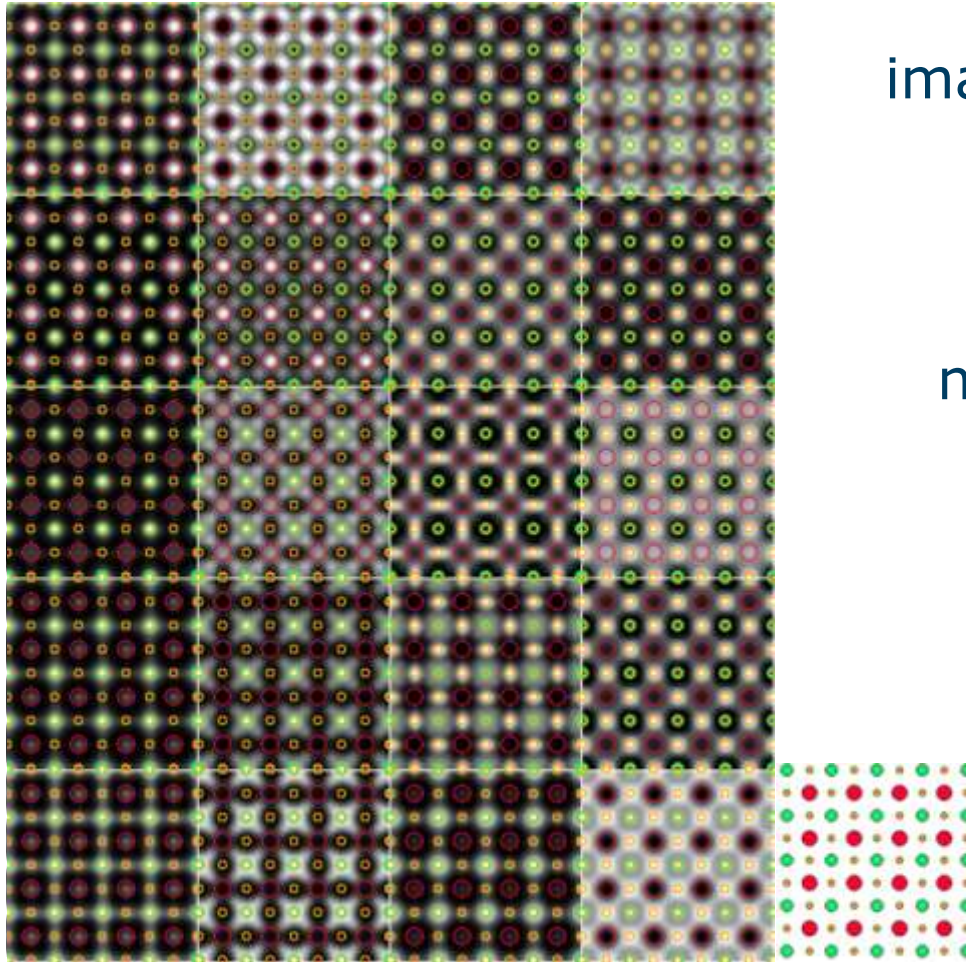


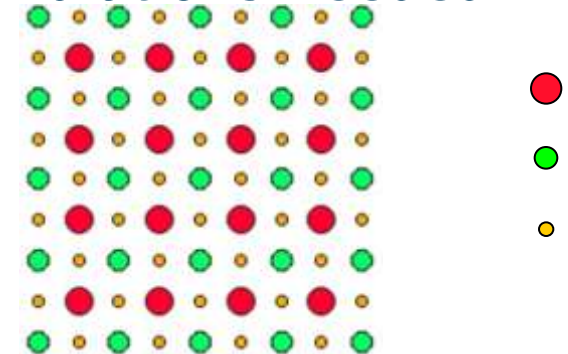
image influenced by contrast transfer function of the microscope



no direct correspondence

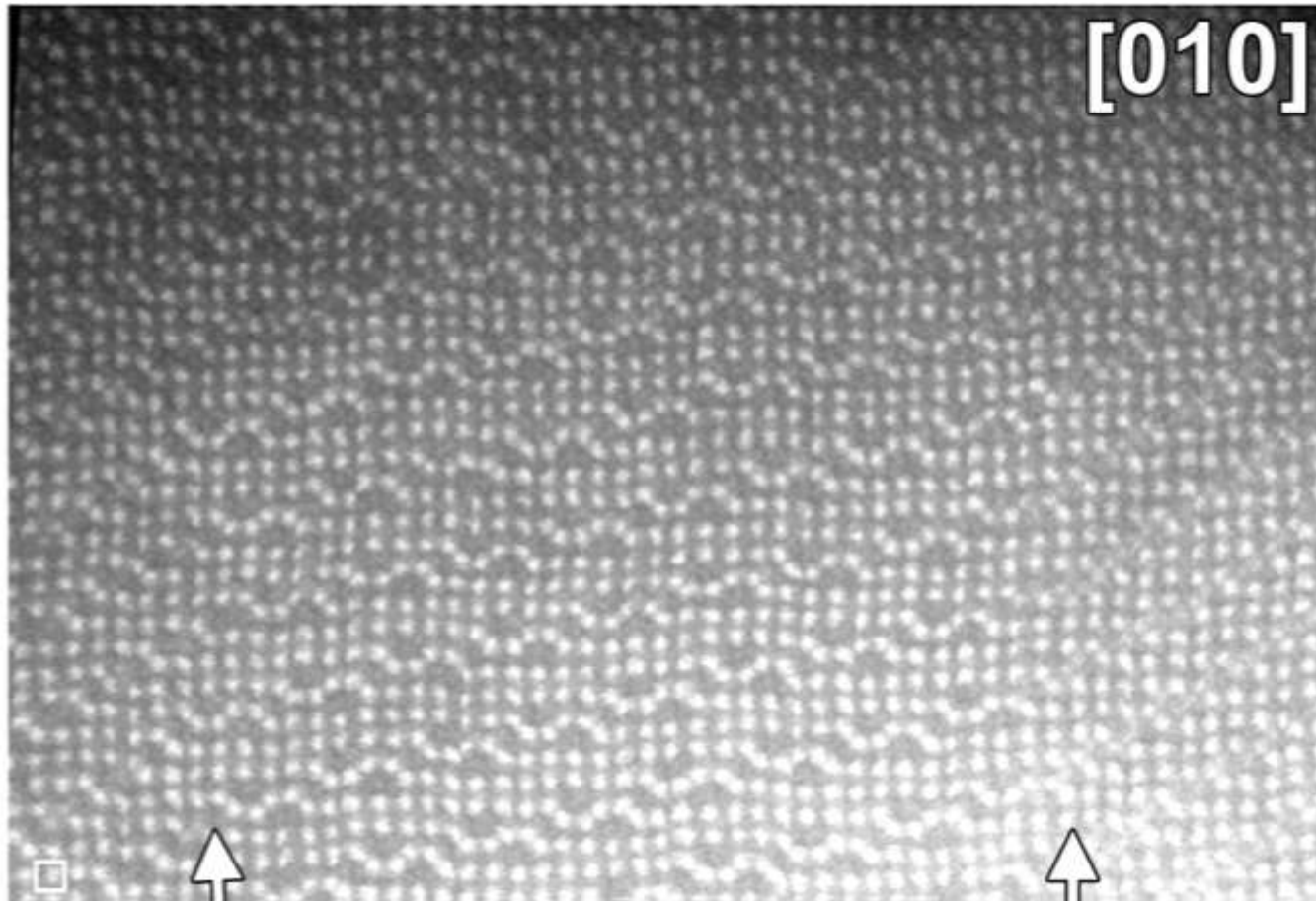


visual comparison with simulations needed



 **simple perovskite**

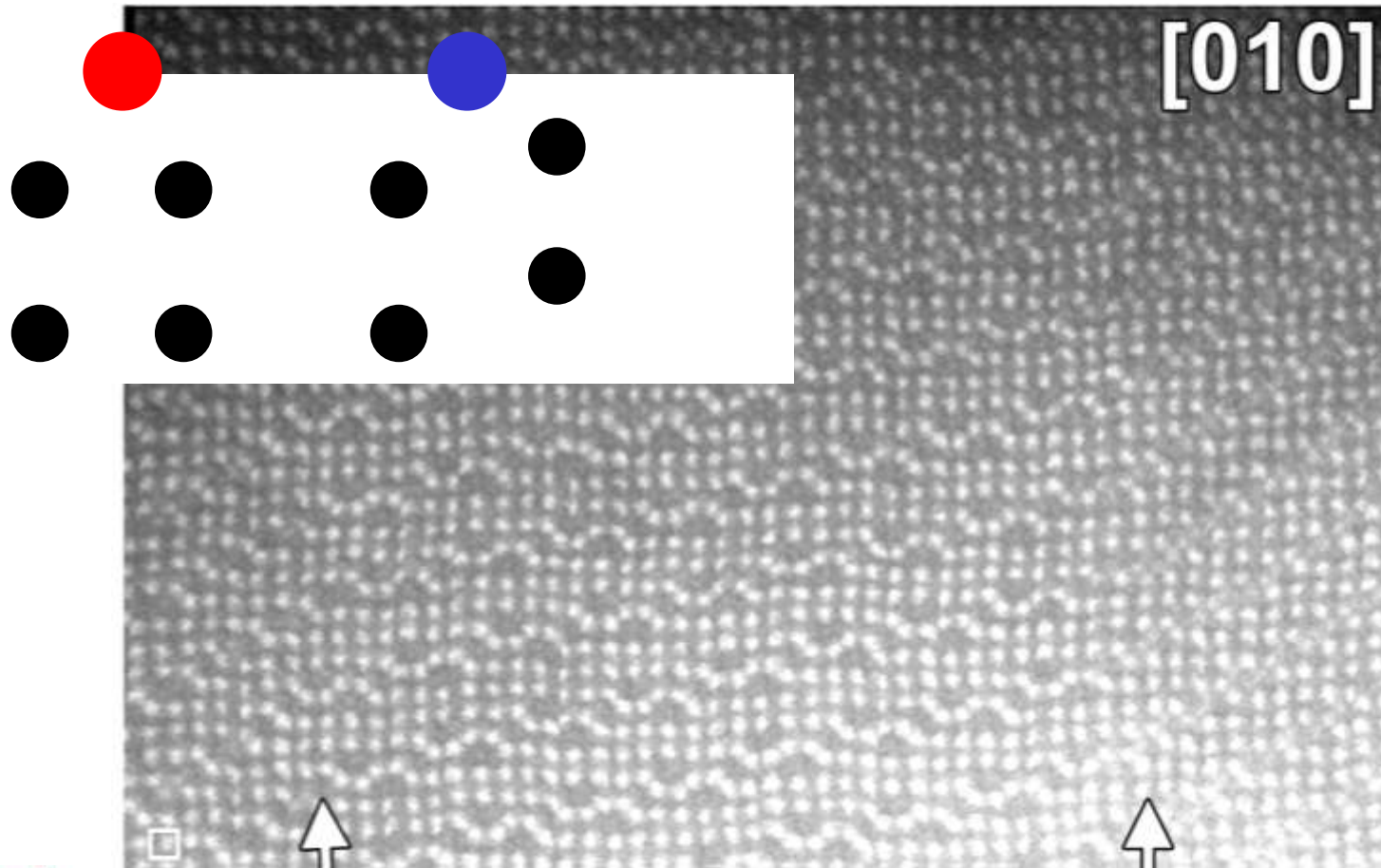
Exercise 3: predict what the ED pattern will roughly look like



Hadermann et al., Solid State Sciences, 10; 4 (2008) 382-389

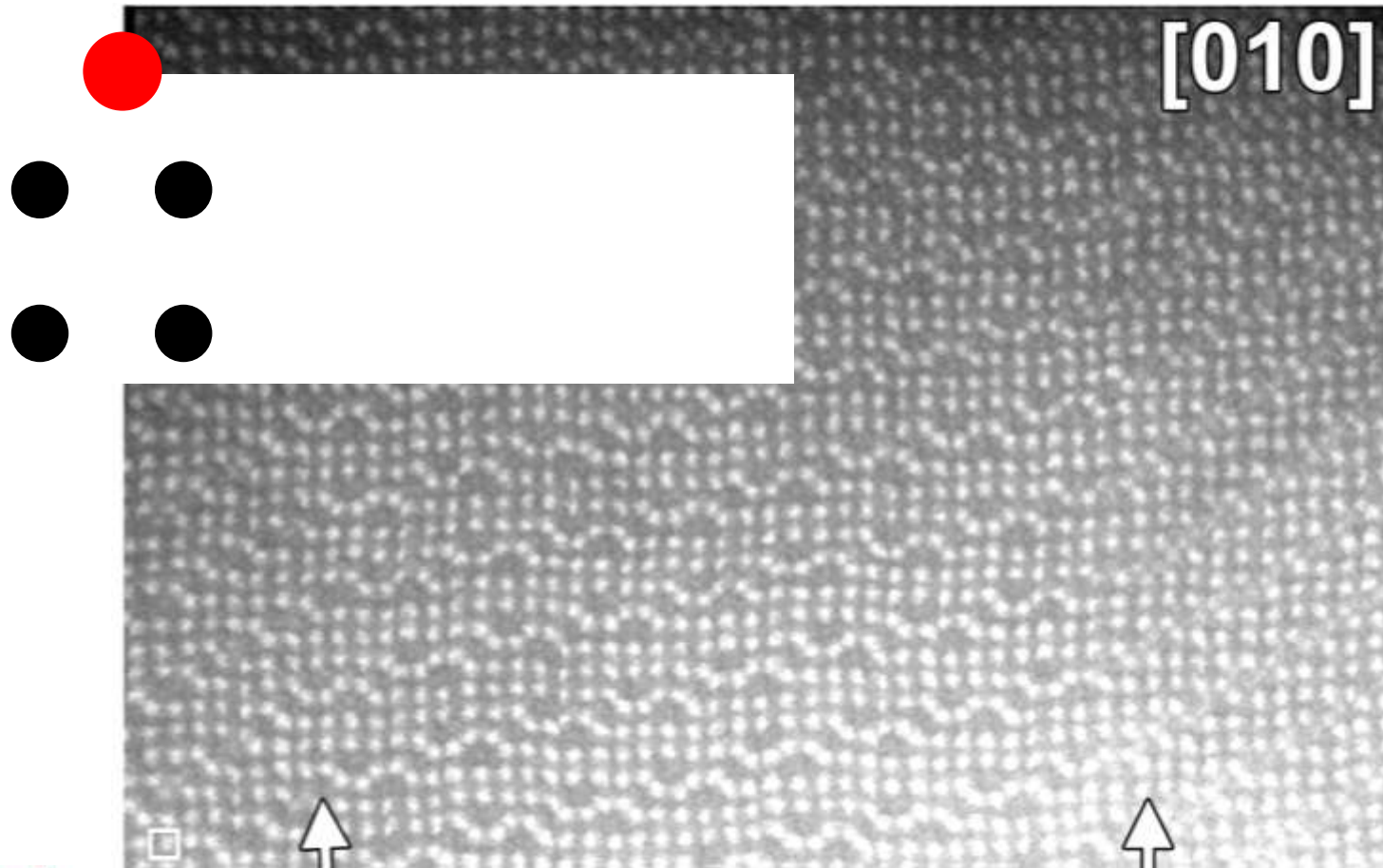
Exercise 3: predict what the ED pattern will roughly look like

What will be the subcell reflections?

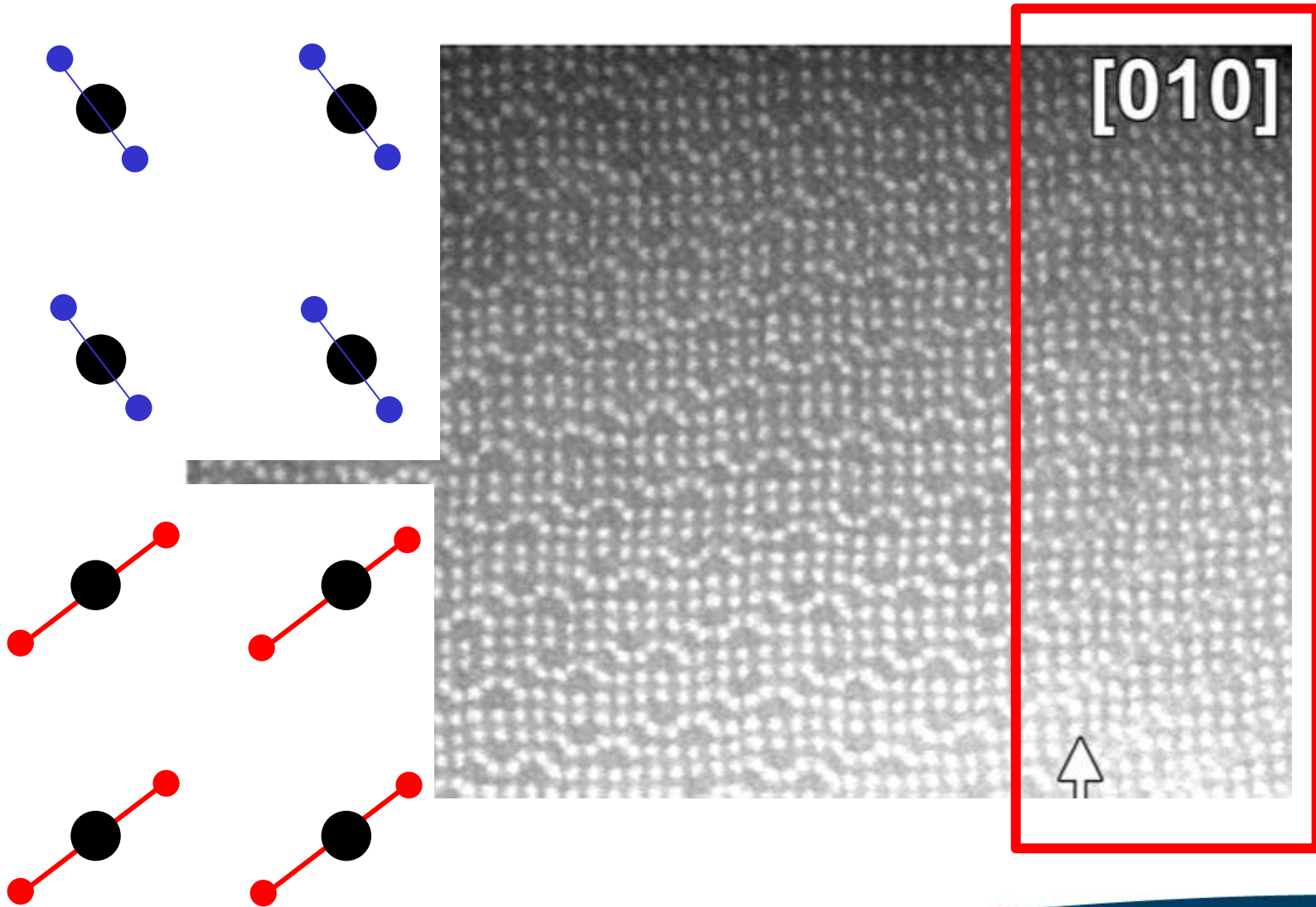


Exercise 3: predict what the ED pattern will roughly look like

What will be the subcell reflections?

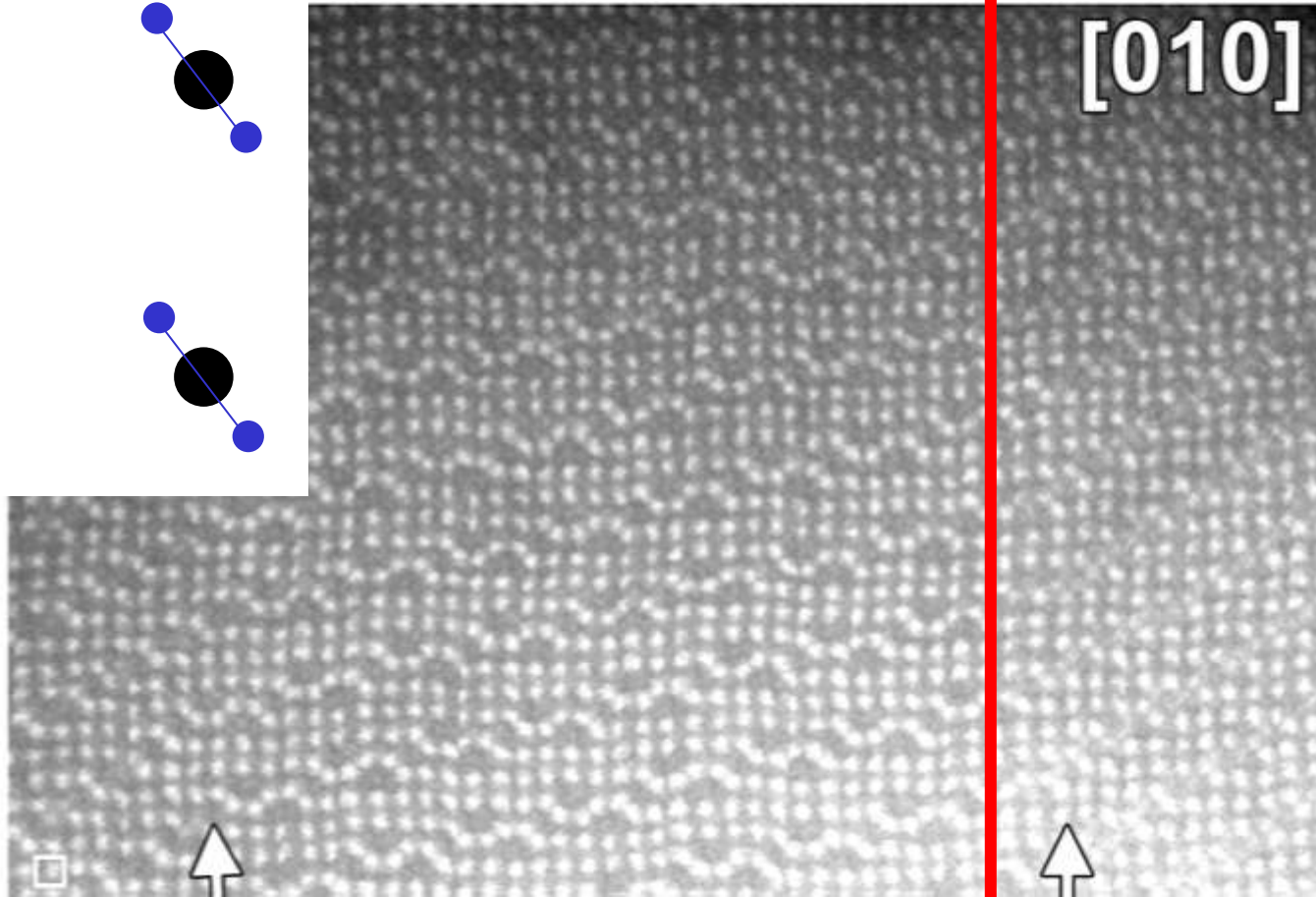
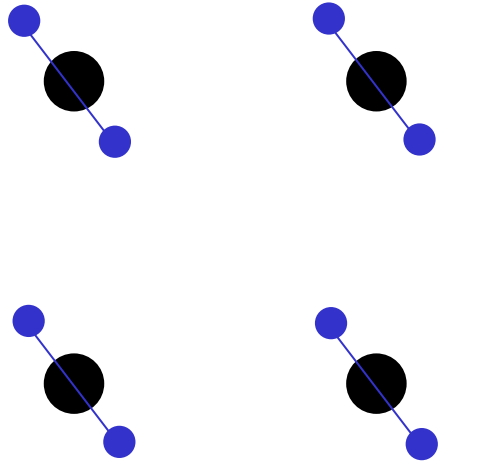


Exercise 3: predict what the ED pattern will roughly look like

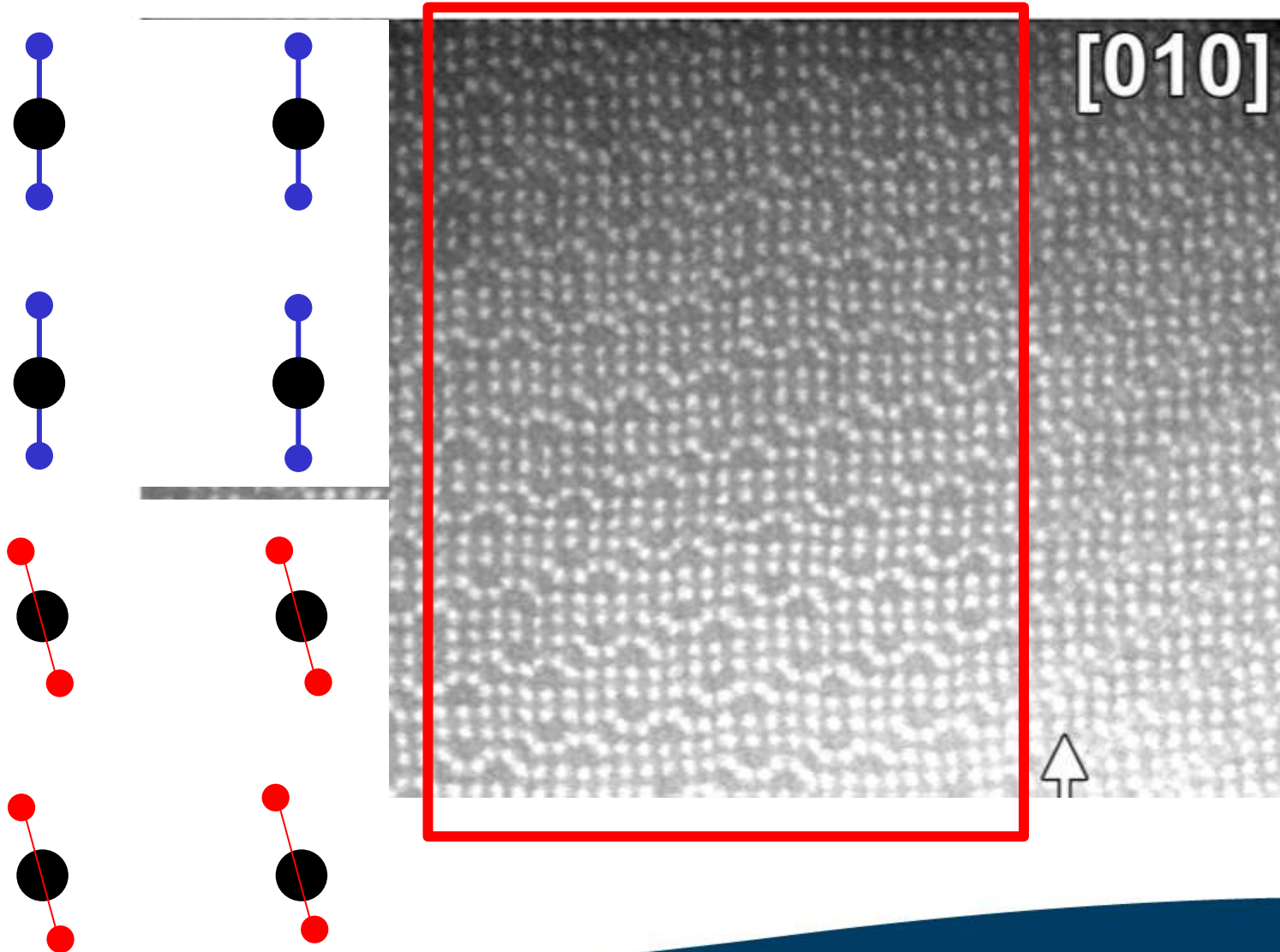


The diagram illustrates the relationship between molecular structure and electron diffraction (ED) patterns. On the left, four molecular models are shown in a 2x2 grid. The top two models have a central black sphere with two blue spheres attached, representing a diatomic molecule. The bottom two models have a central black sphere with two red spheres attached, representing another diatomic molecule. In the center is a grayscale ED pattern showing a regular grid of spots. A red rectangular box highlights a vertical section of the pattern, labeled $[010]$ in the top right corner. A white arrow points upwards from the bottom of this box, indicating the direction of the incident beam.

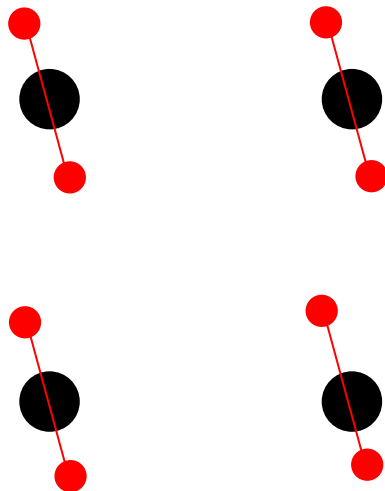
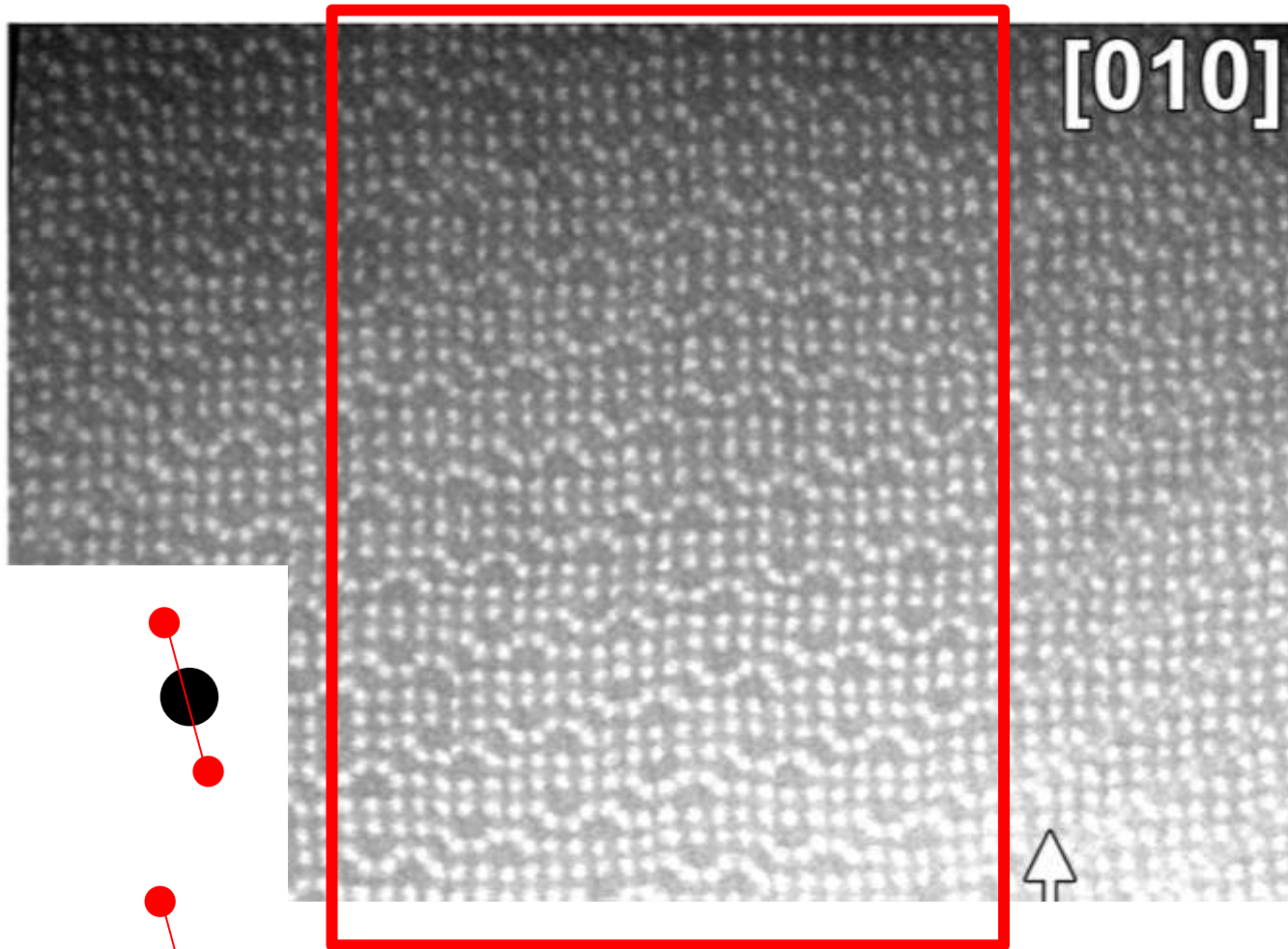
Exercise 3: predict what the ED pattern will roughly look like

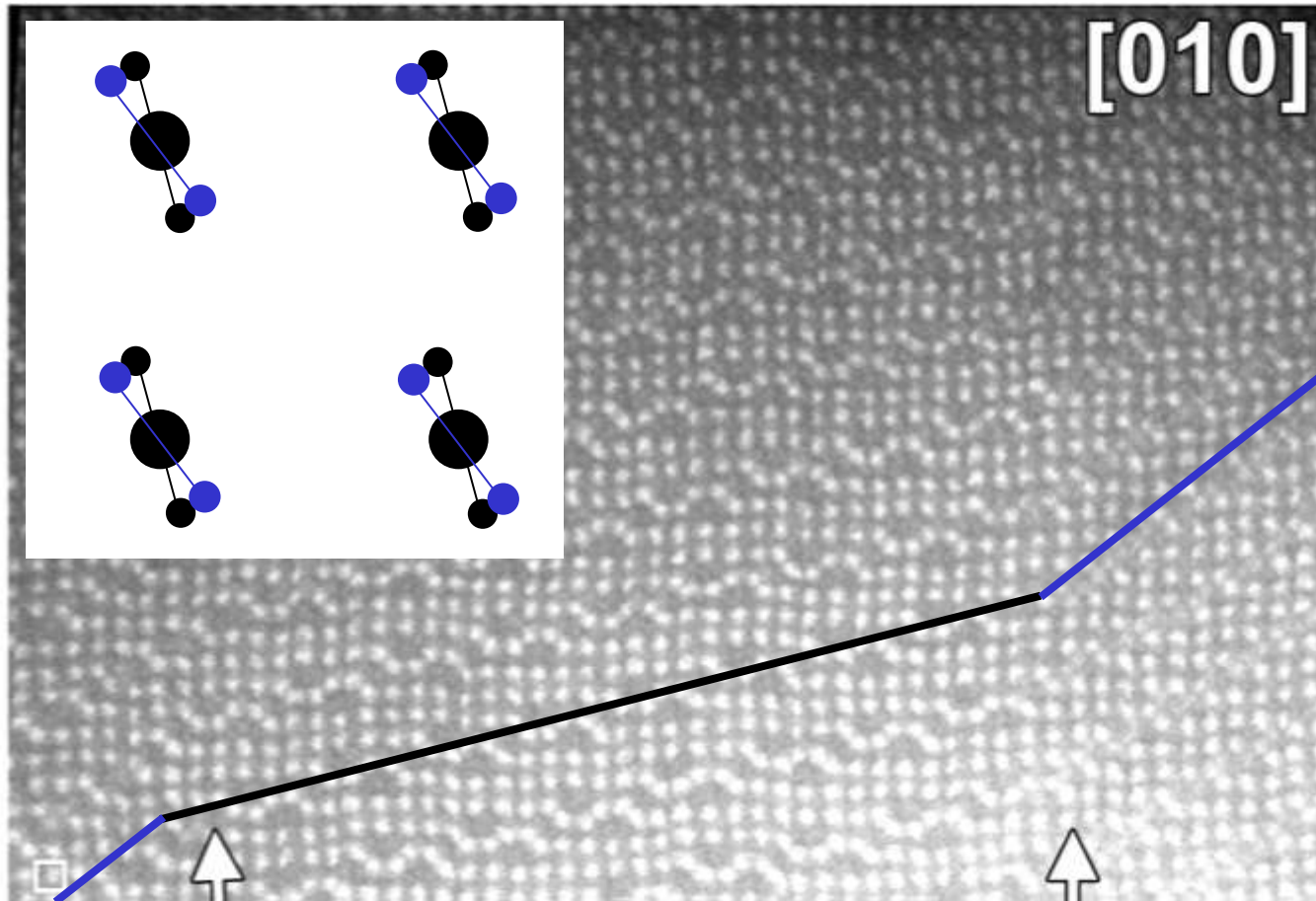


Exercise 3: predict what the ED pattern will roughly look like

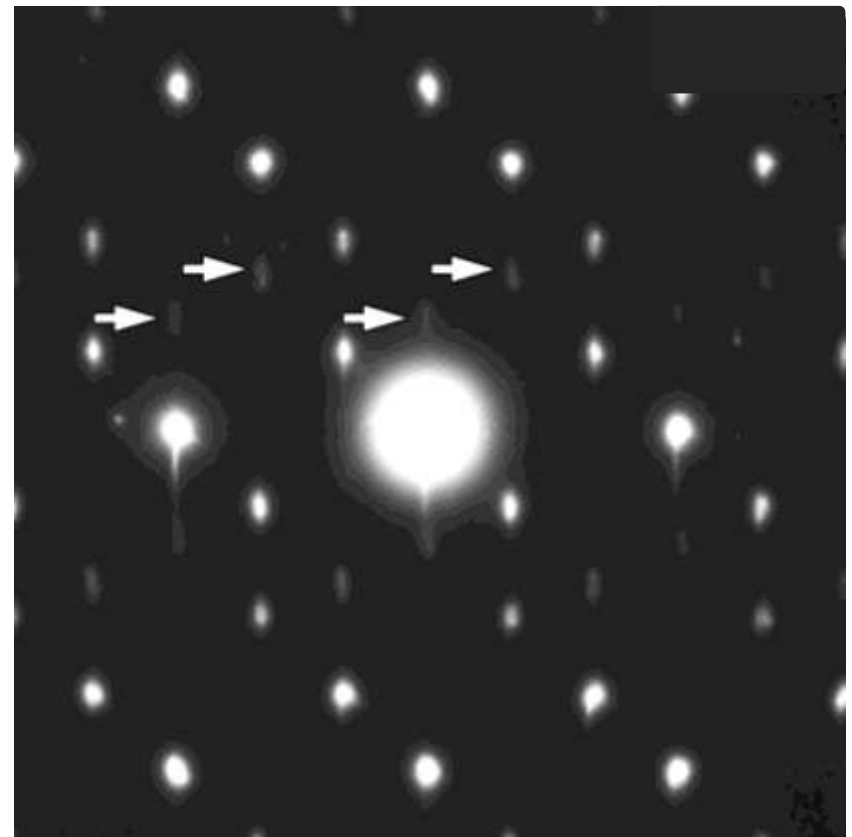
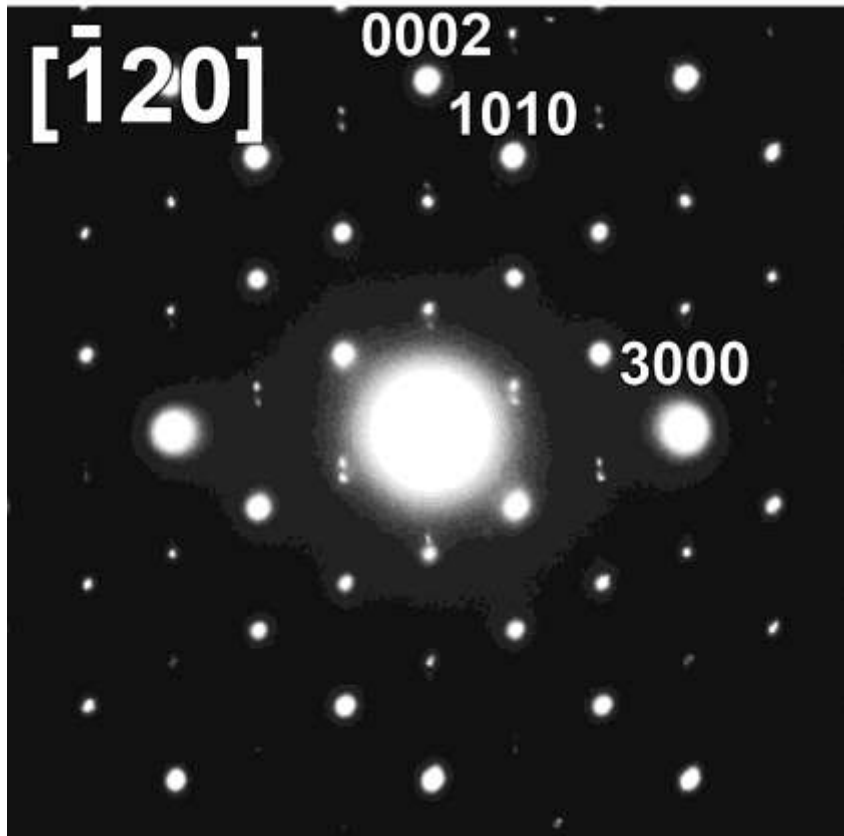


Exercise 3: predict what the ED pattern will roughly look like





Domains with varying q



Mandal et al., Chem. Mater., 19, 25 (2007) 6158



Comments to the applicability of conclusions from TEM



- If the satellites cannot be seen on the XRPD-NPD patterns it is no use to try to refine the structure using the ED cell.
- Precision of parameters determined from ED is not high. ED is good for determining parameters (cell parameters and direction and length of q) qualitatively.
- Good for determining symmetry.
- Can give direct space information and compositional information also.



Purpose of this lecture

At the end of this lecture you should be able to:

- Understand the TEM paragraph in papers about IMS and CS
- Be able to make solid comments about conclusions claimed from TEM by different sources (collaborators, papers,...) by knowing some possible pitfalls
- Decide whether it would be useful to do TEM on your own IMS or CS
- Make basic interpretations of TEM data on your own materials by yourself